SL Paper 1

Let $f(x) = (x-5)^3$, for $x \in \mathbb{R}$.

a. Find $f^{-1}(x)$. [3]

b. Let g be a function so that $(f \circ g)(x) = 8x^6$. Find $g(x)$. [3]

Markscheme

a. interchanging x and y (seen anywhere) ***(M1)***

eg $x = (y-5)^3$

evidence of correct manipulation ***(A1)***

eg $y - 5 = \sqrt[3]{x}$

$f^{-1}(x) = \sqrt[3]{x} + 5 \quad (\text{accept } 5 + x^{\frac{1}{3}},\ y = 5 + \sqrt[3]{x})$ ***A1 N2***

Notes: If working shown, and they do not interchange x and y, award ***A1A1M0*** for $\sqrt[3]{y} + 5$.

If no working shown, award ***N1*** for $\sqrt[3]{y} + 5$.

b. **METHOD 1**

attempt to form composite (in any order) ***(M1)***

eg $g\left((x-5)^3\right),\ (g(x)-5)^3 = 8x^6,\ f(2x^2+5)$

correct working ***(A1)***

eg $g - 5 = 2x^2,\ \left((2x^2+5)-5\right)^3$

$g(x) = 2x^2 + 5$ ***A1 N2***

METHOD 2

recognising inverse relationship ***(M1)***

eg $f^{-1}(8x^6) = g(x),\ f^{-1}(f \circ g)(x) = f^{-1}(8x^6)$

correct working

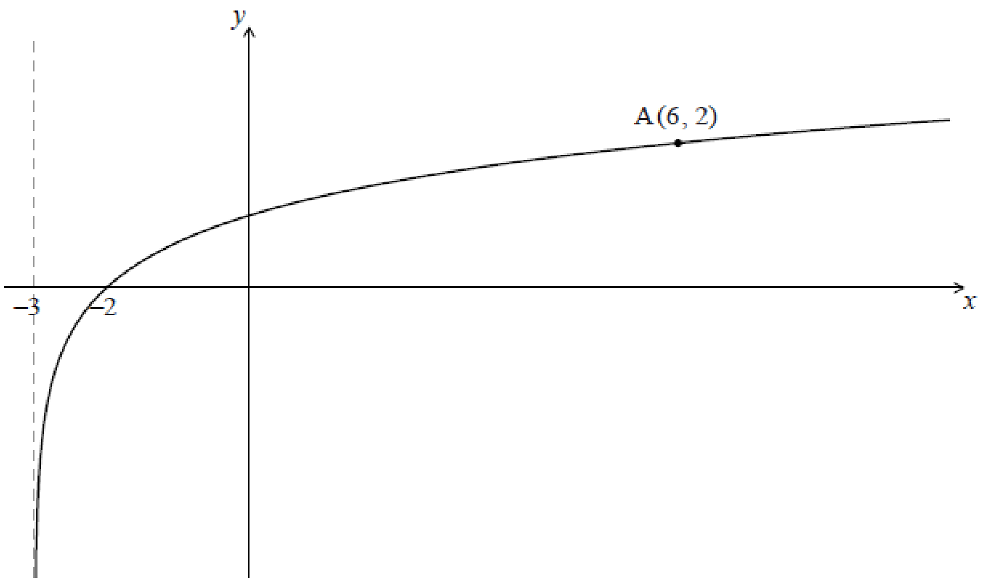
eg $g(x) = \sqrt[3]{(8x^6)} + 5$ ***(A1)***

$g(x) = 2x^2 + 5$ ***A1 N2***

Examiners report

a. [N/A]
b. [N/A]

Let $f(x) = \log_p(x+3)$ for $x > -3$. Part of the graph of f is shown below.



The graph passes through A(6, 2) , has an x-intercept at (−2, 0) and has an asymptote at $x = -3$.

a. Find p . [4]

b. The graph of f is reflected in the line $y = x$ to give the graph of g . [5]

(i) Write down the y-intercept of the graph of g .

(ii) Sketch the graph of g , noting clearly any asymptotes and the image of A.

c. The graph of f is reflected in the line $y = x$ to give the graph of g . [4]

Find $g(x)$.

Markscheme

a. evidence of substituting the point A ***(M1)***

e.g. $2 = \log_p(6+3)$

manipulating logs ***A1***

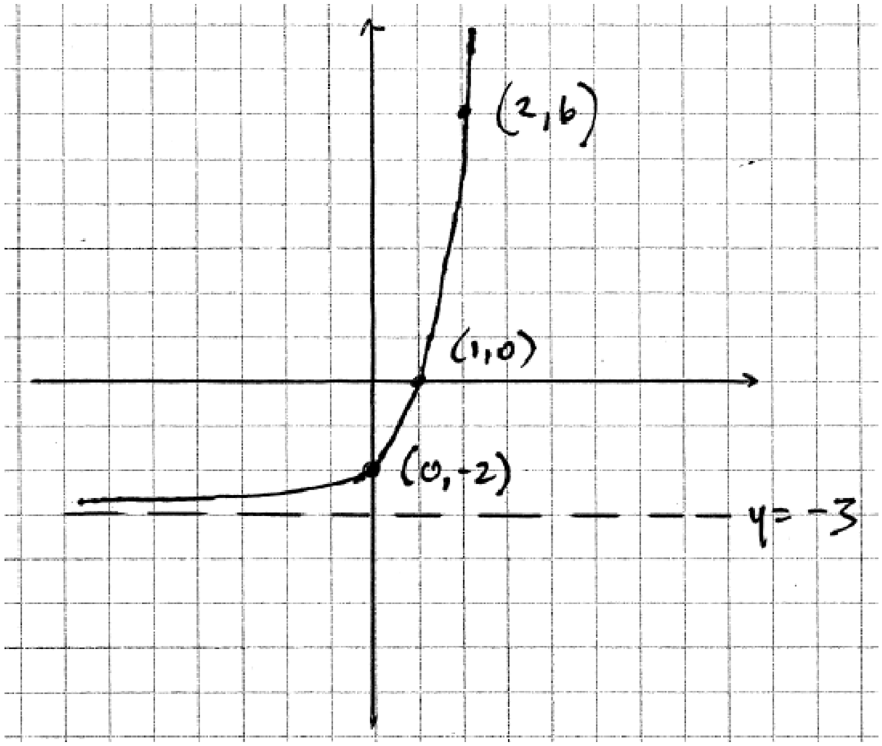
e.g. $p^2 = 9$

$p = 3$ ***A2*** ***N2***

[4 marks]

b. (i) $y = -2$ (accept $(0,\ -2)$) ***A1*** ***N1***

(ii)



A1A1A1A1 *N4*

Note: Award *A1* for asymptote at $y = -3$, *A1* for an increasing function that is concave up, *A1* for a positive *x*-intercept and a negative *y*-intercept, *A1* for passing through the point $(2, 6)$.

[5 marks]

c. **METHOD 1**

recognizing that $g = f^{-1}$ ***(R1)***

evidence of valid approach ***(M1)***

e.g. switching *x* and *y* (seen anywhere), solving for *x*

correct manipulation ***(A1)***

e.g. $3^x = y + 3$

$g(x) = 3^x - 3$ ***A1*** ***N3***

METHOD 2

recognizing that $g(x) = a^x + b$ ***(R1)***

identifying vertical translation ***(A1)***

e.g. graph shifted down 3 units, $f(x) - 3$

evidence of valid approach ***(M1)***

e.g. substituting point to identify the base

$g(x) = 3^x - 3$ ***A1*** ***N3***

[4 marks]

Examiners report

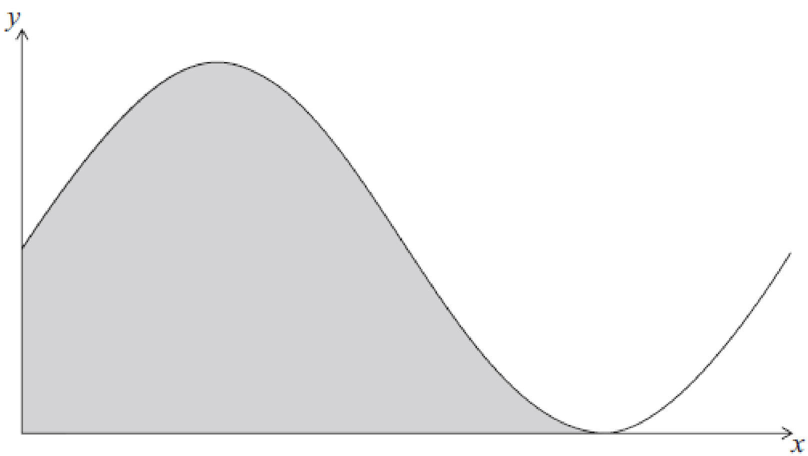
a. In part (a), many candidates successfully substituted the point A to find the base of the logarithm, although some candidates lost a mark for not showing their manipulation of the logarithm equation into the exponential equation.

b. A number of candidates who correctly stated the y-intercept was -2 had difficulty sketching the graph of the reflection in the line $y = x$. A number of candidates graphed directly on the question paper rather than sketching their own graph; candidates should be reminded to show all working for Section B on separate paper. Some correct sketches did not have the position of A indicated. Many candidates had difficulty reflecting the asymptote.

c. Part (c) was often well done, with candidates showing clear and correct working.

The most successful candidates clearly appreciated the linkage between the question parts.

Let $f(x) = 6 + 6\sin x$. Part of the graph of f is shown below.



The shaded region is enclosed by the curve of f , the x-axis, and the y-axis.

a(i) and (ii). Solve for $0 \le x < 2\pi$ [5]

(i) $6 + 6\sin x = 6$;

(ii) $6 + 6\sin x = 0$.

b. Write down the exact value of the x-intercept of f , for $0 \le x < 2\pi$. [1]

c. The area of the shaded region is k . Find the value of k , giving your answer in terms of π . [6]

d. Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of f is transformed to the graph of g. [2]

Give a full geometric description of this transformation.

e. Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of f is transformed to the graph of g. [3]

Given that $\int_p^{p+\frac{3\pi}{2}} g(x)\mathrm{d}x = k$ and $0 \le p < 2\pi$, write down the two values of p.

Markscheme

a(i) and (ii). (i) $\sin x = 0$ ***A1***

$x = 0$, $x = \pi$ ***A1A1*** ***N2***

(ii) $\sin x = -1$ ***A1***

$x = \frac{3\pi}{2}$ ***A1*** ***N1***

[5 marks]

b. $\frac{3\pi}{2}$ ***A1*** ***N1***

[1 mark]

c. evidence of using anti-differentiation ***(M1)***

e.g. $\int_0^{\frac{3\pi}{2}} (6 + 6\sin x)\mathrm{d}x$

correct integral $6x - 6\cos x$ (seen anywhere) ***A1A1***

correct substitution ***(A1)***

e.g. $6\left(\frac{3\pi}{2}\right) - 6\cos\left(\frac{3\pi}{2}\right) - (-6\cos 0)$, $9\pi - 0 + 6$

$k = 9\pi + 6$ ***A1A1*** ***N3***

[6 marks]

d. translation of $\begin{pmatrix} \frac{\pi}{2} \\ 0 \end{pmatrix}$ ***A1A1*** ***N2***

[2 marks]

e. recognizing that the area under *g* is the same as the shaded region in *f* ***(M1)***

$p = \frac{\pi}{2}$, $p = 0$ ***A1A1*** ***N3***

[3 marks]

Examiners report

a(i) and (ii). Many candidates again had difficulty finding the common angles in the trigonometric equations. In part (a), some did not show sufficient working in solving the equations. Others obtained a single solution in (a)(i) and did not find another. Some candidates worked in degrees; the majority worked in radians.

b. While some candidates appeared to use their understanding of the graph of the original function to find the *x*-intercept in part (b), most used their working from part (a)(ii) sometimes with follow-through on an incorrect answer.

c. Most candidates recognized the need for integration in part (c) but far fewer were able to see the solution through correctly to the end. Some did not show the full substitution of the limits, having incorrectly assumed that evaluating the integral at 0 would be 0; without this working, the mark for evaluating at the limits could not be earned. Again, many candidates had trouble working with the common trigonometric values.

d. While there was an issue in the wording of the question with the given domains, this did not appear to bother candidates in part (d). This part was often well completed with candidates using a variety of language to describe the horizontal translation to the right by $\frac{\pi}{2}$.

e. Most candidates who attempted part (e) realized that the integral was equal to the value that they had found in part (c), but a majority tried to integrate the function *g* without success. Some candidates used sketches to find one or both values for *p*. The problem in the wording of the question did not appear to have been noticed by candidates in this part either.

Let $f(x) = x^2$ and $g(x) = 2(x-1)^2$.

a. The graph of g can be obtained from the graph of f using two transformations. [2]

Give a full geometric description of each of the two transformations.

b. The graph of g is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ to give the graph of h. [4]

The point $(-1, 1)$ on the graph of f is translated to the point P on the graph of h.

Find the coordinates of P.

Markscheme

a. in any order

translated 1 unit to the right ***A1*** ***N1***

stretched vertically by factor 2 ***A1*** ***N1***

[2 marks]

b. **METHOD 1**

finding coordinates of image on g ***(A1)(A1)***

e.g. $-1+1=0$, $1 \times 2 = 2$, $(-1, 1) \to (-1+1, 2 \times 1)$, $(0, 2)$

P is (3, 0) ***A1A1*** ***N4***

METHOD 2

$h(x) = 2(x-4)^2 - 2$ ***(A1)(A1)***

P is $(3, 0)$ ***A1A1*** ***N4***

Examiners report

a. The translation was often described well as horizontal (or shift) one unit right. There was considerable difficulty describing the vertical stretch as it was often referred to as "stretch by 2" or "amplitude of 2". A full description should include the name (e.g. vertical stretch) and value for full marks. Candidates also had difficulty applying two consecutive transformations to a single point. Often the translations were applied directly to $(-1, 1)$ instead of first mapping from f to g . A good number of candidates correctly found $h(x)$, but most could not find P from this function.

b. The translation was often described well as horizontal (or shift) one unit right. There was considerable difficulty describing the vertical stretch as it was often referred to as "stretch by 2" or "amplitude of 2". A full description should include the name (e.g. vertical stretch) and value for full marks. Candidates also had difficulty applying two consecutive transformations to a single point. Often the translations were applied directly to $(-1, 1)$ instead of first mapping from f to g . A good number of candidates correctly found $h(x)$, but most could not find P from this function.

Consider $f(x) = x^2 + qx + r$. The graph of f has a minimum value when $x = -1.5$.

The distance between the two zeros of f is 9.

a. Show that the two zeros are 3 and -6. [2]

b. Find the value of q and of r. [4]

Markscheme

a. recognition that the x-coordinate of the vertex is -1.5 (seen anywhere) ***(M1)***

eg axis of symmetry is -1.5, sketch, $f'(-1.5) = 0$

correct working to find the zeroes ***A1***

eg -1.5 ± 4.5

$x = -6$ and $x = 3$ ***AG*** ***N0***

[2 marks]

b. **METHOD 1 (using factors)**

attempt to write factors ***(M1)***

eg $(x-6)(x+3)$

correct factors ***A1***

eg $(x-3)(x+6)$

$q = 3,\ r = -18$ ***A1A1*** ***N3***

METHOD 2 (using derivative or vertex)

valid approach to find q ***(M1)***

eg $f'(-1.5) = 0,\ -\frac{q}{2a} = -1.5$

$q = 3$ ***A1***

correct substitution ***A1***

eg $3^2 + 3(3) + r = 0,\ (-6)^2 + 3(-6) + r = 0$

$r = -18$ ***A1***

$q = 3,\ r = -18$ ***N3***

METHOD 3 (solving simultaneously)

valid approach setting up system of two equations ***(M1)***

eg $9 + 3q + r = 0,\ 36 - 6q + r = 0$

one correct value

eg $q = 3,\ r = -18$ ***A1***

correct substitution ***A1***

eg $3^2 + 3(3) + r = 0,\ (-6)^2 + 3(-6) + r = 0,\ 3^2 + 3q - 18 = 0,\ 36 - 6q - 18 = 0$

second correct value ***A1***

eg $q = 3,\ r = -18$

$q = 3,\ r = -18$ ***N3***

[4 marks]

Examiners report

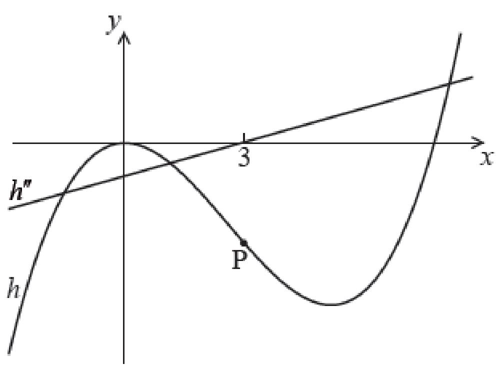
a. As a ‘show that’ question, part a) required a candidate to independently find the answers. Again, too many candidates used the given answers (of 3 and -6) to show that the two zeros were 3 and -6 (a circular argument). Those who were able to recognize that the x-coordinate of the vertex is -1.5 tended to then use the given answers and work backwards thus scoring no further marks in part a).

b. Answers to part b) were more successful with a good variety of methods used and correct solutions seen.

Consider the functions $f(x)$, $g(x)$ and $h(x)$. The following table gives some values associated with these functions.

x	2	3
$f(x)$	2	3
$g(x)$	−14	−18
$f'(x)$	1	1
$g'(x)$	−5	−3
$h''(x)$	−6	0

The following diagram shows parts of the graphs of h and h'' .



There is a point of inflexion on the graph of h at P, when $x = 3$.

Given that $h(x) = f(x) \times g(x)$,

a. Write down the value of $g(3)$, of $f'(3)$, and of $h''(2)$. [3]

b. Explain why P is a point of inflexion. [2]

c. find the y-coordinate of P. [2]

d. find the equation of the normal to the graph of h at P. [7]

Markscheme

a. $g(3) = -18$, $f'(3) = 1$, $h''(2) = -6$ ***A1A1A1*** ***N3***

[3 marks]

b. $h''(3) = 0$ ***(A1)***

valid reasoning ***R1***

eg h'' changes sign at $x = 3$, change in concavity of h at $x = 3$

so P is a point of inflexion ***AG*** ***N0***

[2 marks]

c. writing $h(3)$ as a product of $f(3)$ and $g(3)$ ***A1***

eg $f(3) \times g(3)$, $3 \times (-18)$

$h(3) = -54$ ***A1 N1***

[2 marks]

d. recognizing need to find derivative of h ***(R1)***

eg h' , $h'(3)$

attempt to use the product rule (do **not** accept $h' = f' \times g'$) ***(M1)***

eg $h' = fg' + gf'$, $h'(3) = f(3) \times g'(3) + g(3) \times f'(3)$

correct substitution ***(A1)***

eg $h'(3) = 3(-3) + (-18) \times 1$

$h'(3) = -27$ ***A1***

attempt to find the gradient of the normal ***(M1)***

eg $-\frac{1}{m}$, $-\frac{1}{27}x$

attempt to substitute **their** coordinates and **their** normal gradient into the equation of a line ***(M1)***

eg $-54 = \frac{1}{27}(3) + b$, $0 = \frac{1}{27}(3) + b$, $y + 54 = 27(x - 3)$, $y - 54 = \frac{1}{27}(x + 3)$

correct equation in any form ***A1*** ***N4***

eg $y + 54 = \frac{1}{27}(x - 3)$, $y = \frac{1}{27}x - 54\frac{1}{9}$

[7 marks]

Examiners report

a. Nearly all candidates who attempted to answer parts (a) and (c) did so correctly, as these questions simply required them to understand the notation being used and to read the values from the given table.

b. In part (b), the majority of candidates earned one mark for stating that $h''(x) = 0$ at point P. As this is not enough to determine a point of inflexion, very few candidates earned full marks on this question.

c. Nearly all candidates who attempted to answer parts (a) and (c) did so correctly, as these questions simply required them to understand the notation being used and to read the values from the given table.

d. Part (d) proved to be quite challenging for even the strongest candidates, as almost none of them used the product rule to find $h'(3)$. The most common error was to say $h'(3) = f'(3) \times g'(3)$. Despite this error, many candidates were able to earn further method marks for their work in finding the equation of the normal. There were also a small number of candidates who were able to find the equation for $h'(x)$, and from that

$h''(x)$. These candidates were often successful in earning full marks, although this method was quite time-consuming.

Let $f(x) = 2x - 1$ and $g(x) = 3x^2 + 2$.

a. Find $f^{-1}(x)$. [3]

b. Find $(f \circ g)(1)$. [3]

Markscheme

a. interchanging *x* and *y* (seen anywhere) ***(M1)***

e.g. $x = 2y - 1$

correct manipulation ***(A1)***

e.g. $x + 1 = 2y$

$f^{-1}(x) = \frac{x+1}{2}$ ***A1 N2***

[3 marks]

b. **METHOD 1**

attempt to find or $g(1)$ or $f(1)$ ***(M1)***

$g(1) = 5$ ***(A1)***

$f(5) = 9$ ***A1 N2***

[3 marks]

METHOD 2

attempt to form composite (in any order) ***(M1)***

e.g. $2(3x^2 + 2) - 1$, $3(2x - 1)^2 + 2$

$(f \circ g)(1) = 2(3 \times 1^2 + 2) - 1\ (= 6 \times 1^2 + 3)$ ***(A1)***

$(f \circ g)(1) = 9$ ***A1 N2***

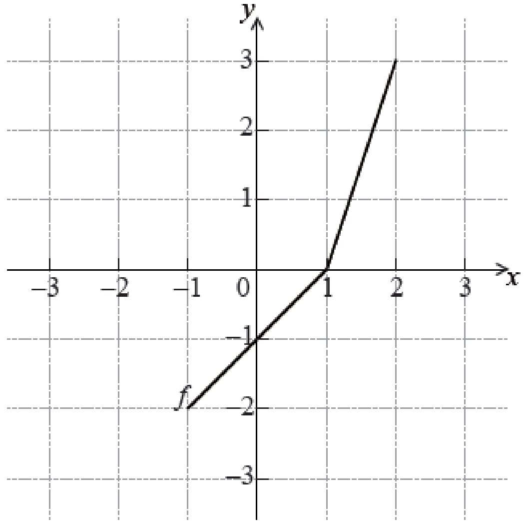
[3 marks]

Examiners report

a. This question was answered correctly by nearly all candidates.

b. This question was answered correctly by nearly all candidates. In part (b), there were a few who seemed unfamiliar with the notation for composition of functions, and attempted to multiply the functions rather than finding the composite, and there were a few who found the correct composite function but failed to substitute in $x = 1$ to find the value.

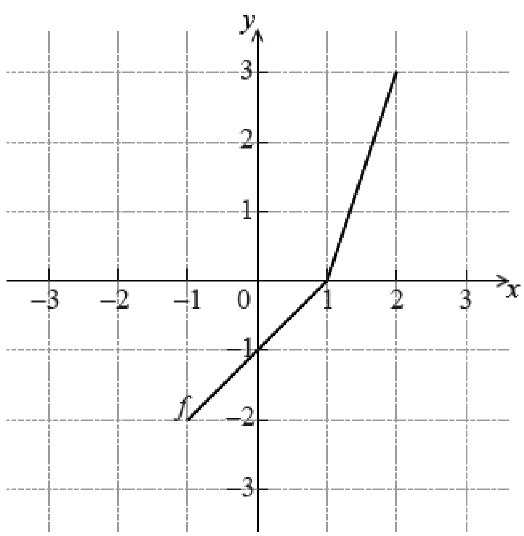
The diagram below shows the graph of a function f , for $-1 \le x \le 2$.



a.i. Write down the value of $f(2)$. [1]

a.ii. Write down the value of $f^{-1}(-1)$. [2]

b. Sketch the graph of f^{-1} on the grid below. [3]



Markscheme

a.i. $f(2) = 3$ ***A1 N1***

[1 mark]

a.ii. $f^{-1}(-1) = 0$ ***A2 N2***

[2 marks]

b. **EITHER**

attempt to draw $y = x$ on grid ***(M1)***

OR

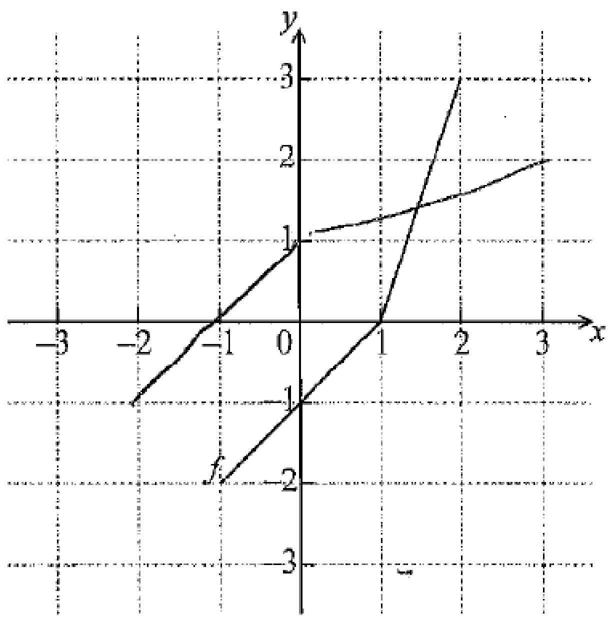
attempt to reverse ***x*** and ***y*** coordinates ***(M1)***

eg writing or plotting **at least two** of the points

$(-2, -1)$, $(-1, 0)$, $(0, 1)$, $(3, 2)$

THEN

correct graph ***A2*** ***N3***



[3 marks]

Examiners report

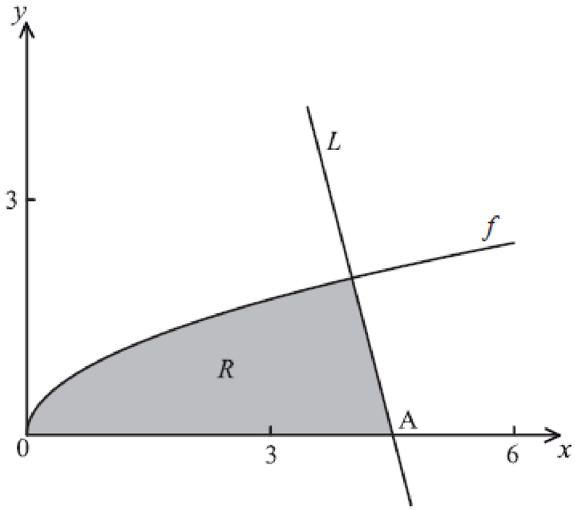
a.i. In part (a) of this question, most candidates were able to find the value of $f(2)$ correctly, while some had trouble finding $f^{-1}(-1)$. Many candidates tried to find an equation for the function, or to make tables of values to help them find their answers. The intent of this question was to read the answers from the given graph. Candidates should be reminded that when the command term is "write down", there is no need for them to do large amounts of working.

a.ii. In part (a) of this question, most candidates were able to find the value of $f(2)$ correctly, while some had trouble finding $f^{-1}(-1)$. Many candidates tried to find an equation for the function, or to make tables of values to help them find their answers. The intent of this question was to read the answers from the given graph. Candidates should be reminded that when the command term is "write down", there is no need for them to do large amounts of working.

b. In part (b) of this question, candidates were generally successful in reversing the x and y coordinates of key points or reflecting in the $y = x$ line to correctly sketch the graph of the inverse function. Common errors included not sketching the graph for the appropriate domain, or sketching the graph of $f(-x)$ or the graph of $-f(x)$.

Let $f(x) = \sqrt{x}$. Line L is the normal to the graph of f at the point (4, 2) .

In the diagram below, the shaded region R is bounded by the x-axis, the graph of f and the line L .



a. Show that the equation of L is $y = -4x + 18$. [4]

b. Point A is the x-intercept of L . Find the x-coordinate of A. [2]

c. Find an expression for the area of R . [3]

d. The region R is rotated 360° about the x-axis. Find the volume of the solid formed, giving your answer in terms of π . [8]

Markscheme

a. finding derivative ***(A1)***

e.g. $f'(x) = \frac{1}{2}x^{\frac{1}{2}}, \frac{1}{2\sqrt{x}}$

correct value of derivative or its negative reciprocal (seen anywhere) ***A1***

e.g. $\frac{1}{2\sqrt{4}}$, $\frac{1}{4}$

gradient of normal = $\frac{1}{\text{gradient of tangent}}$ (seen anywhere) ***A1***

e.g. $-\frac{1}{f'(4)} = -4$, $-2\sqrt{x}$

substituting into equation of line (for normal) ***M1***

e.g. $y - 2 = -4(x - 4)$

$y = -4x + 18$ ***AG*** ***N0***

[4 marks]

b. recognition that $y = 0$ at A ***(M1)***

e.g. $-4x + 18 = 0$

$x = \frac{18}{4} \left(= \frac{9}{2}\right)$ ***A1*** ***N2***

[2 marks]

c. splitting into two appropriate parts (areas and/or integrals) ***(M1)***

correct expression for area of R ***A2*** ***N3***

e.g. area of R = $\int_0^4 \sqrt{x}\mathrm{d}x + \int_4^{4.5} (-4x + 18)\mathrm{d}x$, $\int_0^4 \sqrt{x}\mathrm{d}x + \frac{1}{2} \times 0.5 \times 2$ (triangle)

Note: Award ***A1*** if dx is missing.

[3 marks]

d. correct expression for the volume from $x = 0$ to $x = 4$ ***(A1)***

e.g. $V = \int_0^4 \pi \left[f(x)^2\right] \mathrm{d}x$, $\int_0^4 \pi \sqrt{x}^2 \mathrm{d}x$, $\int_0^4 \pi x \mathrm{d}x$

$V = \left[\frac{1}{2}\pi x^2\right]_0^4$ ***A1***

$V = \pi \left(\frac{1}{2} \times 16 - \frac{1}{2} \times 0\right)$ ***(A1)***

$V = 8\pi$ ***A1***

finding the volume from $x = 4$ to $x = 4.5$

EITHER

recognizing a cone ***(M1)***

e.g. $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(2)^2 \times \frac{1}{2}$ ***(A1)***

$= \frac{2\pi}{3}$ ***A1***

total volume is $8\pi + \frac{2}{3}\pi \left(= \frac{26}{3}\pi\right)$ ***A1*** ***N4***

OR

$V = \pi \int_4^{4.5} (-4x + 18)^2 \mathrm{d}x$ ***(M1)***

$= \int_4^{4.5} \pi(16x^2 - 144x + 324)\mathrm{d}x$

$= \pi \left[\frac{16}{3}x^3 - 72x^2 + 324x\right]_4^{4.5}$ ***A1***

$= \frac{2\pi}{3}$ ***A1***

total volume is $8\pi + \frac{2}{3}\pi \left(= \frac{26}{3}\pi\right)$ ***A1*** ***N4***

[8 marks]

Examiners report

a. Parts (a) and (b) were well done by most candidates.

b. Parts (a) and (b) were well done by most candidates.

c. While quite a few candidates understood that both functions must be used to find the area in part (c), very few were actually able to write a correct expression for this area and this was due to candidates not knowing that they needed to integrate from 0 to 4 and then from 4 to 4.5.

d. On part (d), some candidates were able to earn follow through marks by setting up a volume expression, but most of these expressions were incorrect. If they did not get the expression for the area correct, there was little chance for them to get part (d) correct.

For those candidates who used their expression in part (c) for (d), there was a surprising amount of them who incorrectly applied distributive law of the exponent with respect to the addition or subtraction.

Let $f(x) = 3(x + 1)^2 - 12$.

a. Show that $f(x) = 3x^2 + 6x - 9$. [2]

b(i), (ii) and (iii). For the graph of f [7]

(i) write down the coordinates of the vertex;

(ii) write down the y-intercept;

(iii) find both x-intercepts.

c. **Hence** sketch the graph of f . [3]

d. Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the following two transformations [3]

a stretch of scale factor t in the y-direction,

followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t .

Markscheme

a. $f(x) = 3(x^2 + 2x + 1) - 12$ ***A1***

$= 3x^2 + 6x + 3 - 12$ ***A1***

$= 3x^2 + 6x - 9$ ***AG*** ***N0***

[2 marks]

b(i), (ii) and (iii). (i) vertex is $(-1, -12)$ ***A1A1*** ***N2***

(ii) $y = -9$, or $(0, -9)$ ***A1*** ***N1***

(iii) evidence of solving $f(x) = 0$ ***M1***

e.g. factorizing, formula

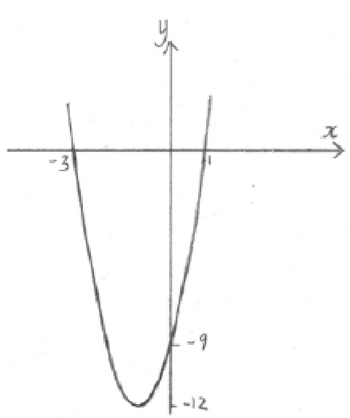
correct working ***A1***

e.g. $3(x+3)(x-1) = 0$, $x = \frac{-6 \pm \sqrt{36+108}}{6}$

$x = -3$, $x = 1$, or $(-3, 0), (1, 0)$ ***A1A1*** ***N2***

[7 marks]

c.



A1A1A1 ***N3***

Note: Award ***A1*** for a parabola opening upward, ***A1*** for vertex in approximately correct position, ***A1*** for intercepts in approximately correct positions. Scale and labelling not required.

[3 marks]

d. $\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -1 \\ -12 \end{pmatrix}$, $t = 3$ ***A1A1A1*** ***N3***

[3 marks]

Examiners report

a. [N/A]

b(i), (ii) and (iii). [N/A]

c. [N/A]

d. [N/A]

Let $f(x) = 7 - 2x$ and $g(x) = x + 3$.

a. Find $(g \circ f)(x)$. [2]

b. Write down $g^{-1}(x)$. [1]

c. Find $(f \circ g^{-1})(5)$. [2]

Markscheme

a. attempt to form composite ***(M1)***

e.g. $g(7 - 2x)$, $7 - 2x + 3$

$(g \circ f)(x) = 10 - 2x$ ***A1*** ***N2***

[2 marks]

b. $g^{-1}(x) = x - 3$ ***A1*** ***N1***

[1 mark]

c. **METHOD 1**

valid approach ***(M1)***

e.g. $g^{-1}(5)$, 2 , $f(5)$

$f(2) = 3$ ***A1*** ***N2***

METHOD 2

attempt to form composite of *f* and g^{-1} ***(M1)***

e.g. $(f \circ g^{-1})(x) = 7 - 2(x - 3)$, $13 - 2x$

$(f \circ g^{-1})(5) = 3$ ***A1*** ***N2***

[2 marks]

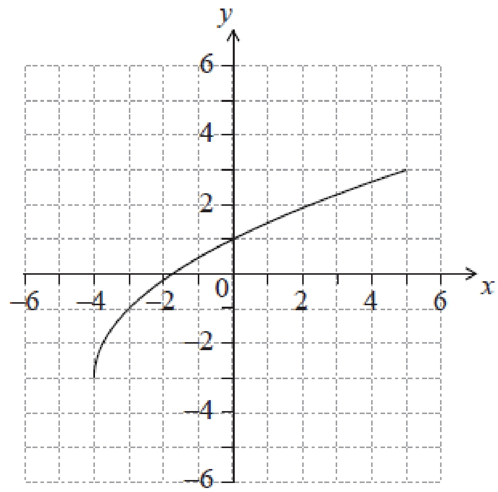
Examiners report

a. A majority of candidates found success in the opening question. Common errors in (a) were to give $f \circ g$ or to multiply *f* by *g*.

b. For (b) some gave the inverse as the reciprocal function $\frac{1}{x+3}$, or wrote $x = y + 3$.

c. Most candidates chose to find a composite in (c), sometimes making simple errors when working with brackets and a negative sign. Only a handful used the more efficient $f(2) = 3$. Additionally, it was not uncommon for candidates to give a correct substitution but not complete the result. Simple expressions such as $(7 - 2x) + 3$ should be finished as $10 - 2x$.

The following diagram shows the graph of $y = f(x)$, for $-4 \leq x \leq 5$.



a(i). Write down the value of $f(-3)$. [1]

a(ii). Write down the value of $f^{-1}(1)$. [1]

b. Find the domain of f^{-1}. [2]

c. On the grid above, sketch the graph of f^{-1}. [3]

Markscheme

a(i). $f(-3) = -1$ ***A1*** ***N1***

[1 mark]

a(ii). $f^{-1}(1) = 0$ (accept $y = 0$) ***A1*** ***N1***

[1 mark]

b. domain of f^{-1} is range of \(f\) ***(R1)***

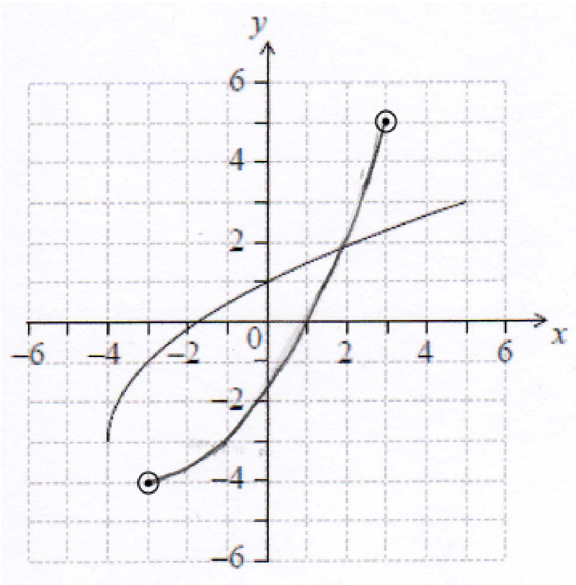
eg $\mathrm{R}f = \mathrm{D}f^{-1}$

correct answer ***A1*** ***N2***

eg $-3 \leqslant x \leqslant 3,\ x \in [-3,\ 3]$ (accept $-3 < x < 3,\ -3 \leqslant y \leqslant 3$)

[2 marks]

c.



A1A1 *N2*

Note: Graph must be approximately correct reflection in $y = x$.

Only if the shape is approximately correct, award the following:

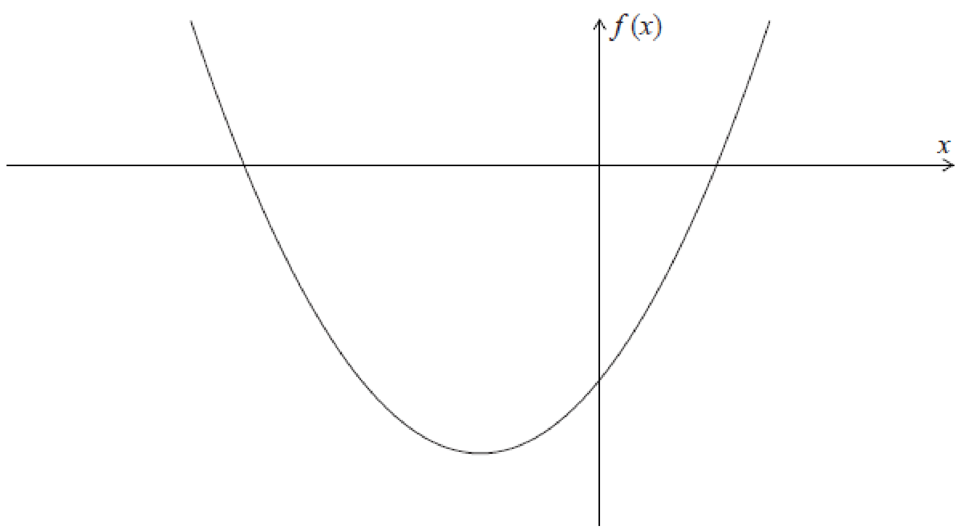
A1 for x-intercept at 1, and *A1* for endpoints within circles.

[2 marks]

Examiners report

a(i). [N/A]
a(ii). [N/A]
b. [N/A]
c. [N/A]

The diagram below shows part of the graph of $f(x) = (x - 1)(x + 3)$.



. (a) Write down the x-intercepts of the graph of f . [6]

(b) Find the coordinates of the vertex of the graph of f .

a. Write down the x-intercepts of the graph of f . [2]

b. Find the coordinates of the vertex of the graph of f . [4]

Markscheme

. (a) $x = 1$, $x = -3$ (accept (1, 0), $(-3, 0)$) ***A1A1*** ***N2***

[2 marks]

(b) **METHOD 1**

attempt to find x-coordinate ***(M1)***

eg $\frac{1+-3}{2}$, $x = \frac{-b}{2a}$, $f'(x) = 0$

correct value, $x = -1$ (may be seen as a coordinate in the answer) ***A1***

attempt to find **their** y-coordinate ***(M1)***

eg $f(-1)$, -2×2 , $y = \frac{-D}{4a}$

$y = -4$ ***A1***

vertex $(-1, -4)$ ***N3***

METHOD 2

attempt to complete the square ***(M1)***

eg $x^2 + 2x + 1 - 1 - 3$

attempt to put into vertex form ***(M1)***

eg $(x + 1)^2 - 4$, $(x - 1)^2 + 4$

vertex $(-1, -4)$ ***A1A1*** ***N3***

[4 marks]

a. $x = 1$, $x = -3$ (accept (1, 0), $(-3, 0)$) ***A1A1*** ***N2***

[2 marks]

b. **METHOD 1**

attempt to find x-coordinate ***(M1)***

eg $\frac{1+-3}{2}$, $x = \frac{-b}{2a}$, $f'(x) = 0$

correct value, $x = -1$ (may be seen as a coordinate in the answer) ***A1***

attempt to find **their** y-coordinate ***(M1)***

eg $f(-1)$, -2×2 , $y = \frac{-D}{4a}$

$y = -4$ ***A1***

vertex $(-1, -4)$ ***N3***

METHOD 2

attempt to complete the square ***(M1)***

eg $x^2 + 2x + 1 - 1 - 3$

attempt to put into vertex form *(M1)*

eg $(x+1)^2-4$, $(x-1)^2+4$

vertex $(-1, -4)$ ***A1A1 N3***

[4 marks]

Examiners report

. Most candidates recognized the values of the x-intercepts from the factorized form of the function. Candidates also showed little difficulty finding the vertex of the graph, and employed a variety of techniques: averaging x-intercepts, using $x = \frac{-b}{2a}$, completing the square.

a. Most candidates recognized the values of the x-intercepts from the factorized form of the function. Candidates also showed little difficulty finding the vertex of the graph, and employed a variety of techniques: averaging x-intercepts, using $x = \frac{-b}{2a}$, completing the square.

b. Most candidates recognized the values of the x-intercepts from the factorized form of the function. Candidates also showed little difficulty finding the vertex of the graph, and employed a variety of techniques: averaging x-intercepts, using $x = \frac{-b}{2a}$, completing the square.

Let $f(x) = \sqrt{x+2}$ for $x \geq 2$ and $g(x) = 3x - 7$ for $x \in \mathbb{R}$.

a. Write down $f(14)$. [1]

b. Find $(g \circ f)$ (14). [2]

c. Find $g^{-1}(x)$. [3]

Markscheme

a. $f(14) = 4$ ***A1 N1***

[1 mark]

b. attempt to substitute ***(M1)***

eg $g(4)$, $3 \times 4 - 7$

5 ***A1 N2***

[2 marks]

c. interchanging x and y (seen anywhere) ***(M1)***

eg $x = 3y - 7$

evidence of correct manipulation ***(A1)***

eg $x + 7 = 3y$

$g^{-1}(x) = \frac{x+7}{3}$ ***A1 N3***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Let $f(x) = 8x + 3$ and $g(x) = 4x$, for $x \in \mathbb{R}$.

a. Write down $g(2)$. [1]

b. Find $(f \circ g)(x)$. [2]

c. Find $f^{-1}(x)$. [2]

Markscheme

a. $g(2) = 8$ ***A1 N1***

[1 mark]

b. attempt to form composite (in any order) ***(M1)***

eg $f(4x),\ 4 \times (8x + 3)$

$(f \circ g)(x) = 32x + 3$ ***A1 N2***

[2 marks]

c. interchanging x and y (may be seen at any time) ***(M1)***

eg $x = 8y + 3$

$f^{-1}(x) = \frac{x-3}{8}$ $\left(\text{accept } \frac{x-3}{8},\ y = \frac{x-3}{8}\right)$ ***A1 N2***

[2 marks]

Examiners report

a. This question was successfully answered by most candidates. The inverse notation was sometimes mistakenly interpreted as derivative or reciprocal.

b. This question was successfully answered by most candidates. The inverse notation was sometimes mistakenly interpreted as derivative or reciprocal.

c. This question was successfully answered by most candidates. The inverse notation was sometimes mistakenly interpreted as derivative or reciprocal.

Let $f(x) = 4x - 2$ and $g(x) = -2x^2 + 8$.

a. Find $f^{-1}(x)$. [3]

b. Find $(f \circ g)(1)$. [3]

Markscheme

a. interchanging x and y (seen anywhere) ***(M1)***

eg $x = 4y - 2$

evidence of correct manipulation ***(A1)***

eg $x + 2 = 4y$

$f^{-1}(x) = \frac{x+2}{4}$ (accept $y = \frac{x+2}{4}$, $\frac{x+2}{4}$, $f^{-1}(x) = \frac{1}{4}x + \frac{1}{2}$ ***A1 N2***

[3 marks]

b. **METHOD 1**

attempt to substitute 1 into $g(x)$ ***(M1)***

eg $g(1) = -2 \times 1^2 + 8$

$g(1) = 6$ ***(A1)***

$f(6) = 22$ ***A1 N3***

METHOD 2

attempt to form composite function (in any order) ***(M1)***

eg $(f \circ g)(x) = 4(-2x^2 + 8) - 2$ $(= -8x^2 + 30)$

correct substitution

eg $(f \circ g)(1) = 4(-2 \times 1^2 + 8) - 2$, $-8 + 30$

$f(6) = 22$ ***A1 N3***

[3 marks]

Examiners report

a. The overwhelming majority of candidates answered both parts of this question correctly. There were a few who seemed unfamiliar with the inverse notation and answered part (a) with the derivative or the reciprocal of the function.

b. The overwhelming majority of candidates answered both parts of this question correctly. A few candidates made arithmetic errors in part (b) which kept them from finding the correct answer.

Let $f(x) = 3x^2 - 6x + p$. The equation $f(x) = 0$ has two equal roots.

a(i). Write down the **value** of the discriminant. [2]

a(ii) Hence, show that $p = 3$. [1]

b. The graph of $f$has its vertex on the x-axis. [4]

Find the coordinates of the vertex of the graph of f.

c. The graph of f has its vertex on the x-axis. [1]

Write down the solution of $f(x) = 0$.

d(i). The graph of f has its vertex on the x-axis. [1]

The function can be written in the form $f(x) = a(x-h)^2 + k$. Write down the value of a.

d(ii). The graph of f has its vertex on the x-axis. [1]

The function can be written in the form $f(x) = a(x-h)^2 + k$. Write down the value of h.

d(iii). The graph of f has its vertex on the x-axis. [1]

The function can be written in the form $f(x) = a(x-h)^2 + k$. Write down the value of k.

e. The graph of f has its vertex on the x-axis. [4]

The graph of a function g is obtained from the graph of f by a reflection of f in the x-axis, followed by a translation by the vector $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

Find g, giving your answer in the form $g(x) = Ax^2 + Bx + C$.

Markscheme

a(i). correct value 0, or $36 - 12p$ ***A2*** ***N2***

[2 marks]

a(ii). correct equation which clearly leads to $p = 3$ ***A1***

eg $36 - 12p = 0,\ 36 = 12p$

$p = 3$ ***AG*** ***N0***

[1 mark]

b. **METHOD 1**

valid approach ***(M1)***

eg $x = -\frac{b}{2a}$

correct working ***A1***

eg $-\frac{(-6)}{2(3)},\ x = \frac{6}{6}$

correct answers ***A1A1*** ***N2***

eg $x = 1,\ y = 0;\ (1,\ 0)$

METHOD 2

valid approach ***(M1)***

eg $f(x) = 0$, factorisation, completing the square

correct working ***A1***

eg $x^2 - 2x + 1 = 0,\ (3x-3)(x-1),\ f(x) = 3(x-1)^2$

correct answers ***A1A1*** ***N2***

eg $x = 1,\ y = 0;\ (1,\ 0)$

METHOD 3

valid approach using derivative ***(M1)***

eg $f'(x) = 0,\ 6x - 6$

correct equation ***A1***

eg $6x - 6 = 0$

correct answers ***A1A1*** ***N2***

eg $x = 1,\ y = 0;\ (1,\ 0)$

[4 marks]

c. $x = 1$ ***A1*** ***N1***

[1 mark]

d(i). $a = 3$ ***A1*** ***N1***

[1 mark]

d(ii). $h = 1$ ***A1*** ***N1***

[1 mark]

d(iii). $k = 0$ ***A1 N1***

[1 mark]

e. attempt to apply vertical reflection ***(M1)***

eg $-f(x),\ -3(x-1)^2$, sketch

attempt to apply vertical shift 6 units up ***(M1)***

eg $-f(x)+6$, vertex $(1, 6)$

transformations performed correctly (in correct order) ***(A1)***

eg $-3(x-1)^2+6,\ -3x^2+6x-3+6$

$g(x) = -3x^2+6x+3$ ***A1 N3***

[4 marks]

Examiners report

a(i). [N/A]
a(ii). [N/A]
b. [N/A]
c. [N/A]
d(i). [N/A]
d(ii). [N/A]
d(iii). [N/A]
e. [N/A]

Consider a function f. The line L_1 with equation $y = 3x+1$ is a tangent to the graph of f when $x = 2$

Let $g(x) = f(x^2+1)$ and P be the point on the graph of g where $x = 1$.

a.i. Write down $f'(2)$. [2]

a.ii. Find $f(2)$. [2]

b. Show that the graph of *g* has a gradient of 6 at P. [5]

c. Let L_2 be the tangent to the graph of *g* at P. L_1 intersects L_2 at the point Q. [7]

Find the y-coordinate of Q.

Markscheme

a.i. recognize that $f'(x)$ is the gradient of the tangent at x ***(M1)***

eg $f'(x) = m$

$f'(2) = 3$ (accept *m* = 3) ***A1 N2***

[2 marks]

a.ii. recognize that $f(2) = y(2)$ ***(M1)***

eg $f(2) = 3 \times 2 + 1$

$f(2) = 7$ ***A1 N2***

[2 marks]

b. recognize that the gradient of the graph of *g* is $g'(x)$ ***(M1)***

choosing chain rule to find $g'(x)$ ***(M1)***

eg $\frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x},\ u = x^2 + 1,\ u' = 2x$

$g'(x) = f'\left(x^2 + 1\right) \times 2x$ ***A2***

$g'(1) = 3 \times 2$ ***A1***

$g'(1) = 6$ ***AG N0***

[5 marks]

c. at Q, $L_1 = L_2$ (seen anywhere) ***(M1)***

recognize that the gradient of L_2 is *g*'(1) (seen anywhere) ***(M1)***
eg *m* = 6

finding *g*(1) (seen anywhere) ***(A1)***
eg $g(1) = f(2),\ g(1) = 7$

attempt to substitute gradient and/or coordinates into equation of a straight line ***M1***
eg $y - g(1) = 6(x - 1),\ y - 1 = g'(1)(x - 7),\ 7 = 6(1) + \mathrm{b}$

correct equation for L_2

eg $y - 7 = 6(x - 1),\ y = 6x + 1$ ***A1***

correct working to find Q ***(A1)***
eg same *y*-intercept, $3x = 0$

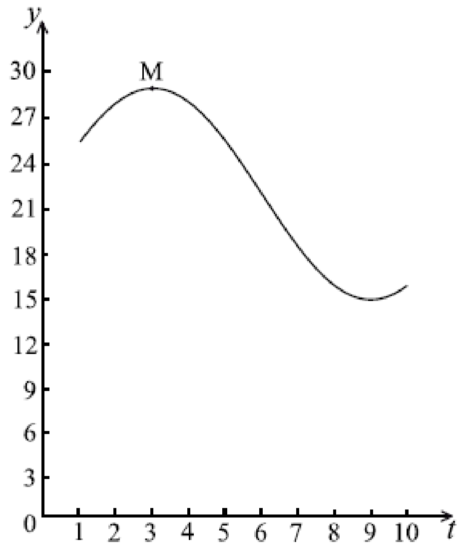
$y = 1$ ***A1 N2***

[7 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]
c. [N/A]

Let $f(t) = a\cos b(t - c) + d$, $t \geq 0$. Part of the graph of $y = f(t)$ is given below.



When $t = 3$, there is a maximum value of 29, at M.

When $t = 9$, there is a minimum value of 15.

a(i), (ii), (iii) and (iv). (i) Find the value of a. [7]

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of d.

(iv) Write down a value for c.

b. The transformation P is given by a horizontal stretch of a scale factor of $\frac{1}{2}$, followed by a translation of $\begin{pmatrix} 3 \\ -10 \end{pmatrix}$. [2]

Let M' be the image of M under P. Find the coordinates of M' .

c. The graph of g is the image of the graph of f under P. [4]

Find $g(t)$ in the form $g(t) = 7\cos B(t - c) + D$.

d. The graph of g is the image of the graph of f under P. [3]

Give a full geometric description of the transformation that maps the graph of g to the graph of f .

Markscheme

a(i), (ii), (iii) and (iv). (i) attempt to substitute ***(M1)***

e.g. $a = \frac{29-15}{2}$

$a = 7$ (accept $a = -7$) ***A1*** ***N2***

(ii) period $= 12$ ***(A1)***

$b = \frac{2\pi}{12}$ ***A1***

$b = \frac{\pi}{6}$ ***AG*** ***N0***

(iii) attempt to substitute ***(M1)***

e.g. $d = \frac{29+15}{2}$

$d = 22$ ***A1*** ***N2***

(iv) $c = 3$ (accept $c = 9$ from $a = -7$) ***A1*** ***N1***

Note: Other correct values for c can be found, $c = 3 \pm 12k$, $k \in \mathbb{Z}$.

[7 marks]

b. stretch takes 3 to 1.5 ***(A1)***

translation maps (1.5, 29) to (4.5, 19) (so M' is (4.5, 19)) ***A1 N2***

[2 marks]

c. $g(t) = 7\cos\frac{\pi}{3}(t - 4.5) + 12$ ***A1A2A1 N4***

Note: Award ***A1*** for $\frac{\pi}{3}$, ***A2*** for 4.5, ***A1*** for 12.

Other correct values for c can be found, $c = 4.5 \pm 6k$, $k \in \mathbb{Z}$.

[4 marks]

d. translation $\begin{pmatrix} -3 \\ 10 \end{pmatrix}$ ***(A1)***

horizontal stretch of a scale factor of 2 ***(A1)***

completely correct description, in correct order ***A1 N3***

e.g. translation $\begin{pmatrix} -3 \\ 10 \end{pmatrix}$ then horizontal stretch of a scale factor of 2

[3 marks]

Examiners report

a(i), (ii), (iii) and (iv). This question was the most difficult on the paper. Where candidates attempted this question, part (a) was answered satisfactorily.

b. Few answered part (b) correctly as most could not interpret the horizontal stretch.

c. Few answered part (b) correctly as most could not interpret the horizontal stretch. As a result, there were many who were unable to answer part (c) although follow through marks were often obtained from incorrect answers in both parts (a) and (b). The link between the answer in (b) and the value of C in part (c) was lost on all but the most attentive.

d. In part (d), some candidates could name the transformations required, although only a handful provided the correct order of the transformations to return the graph to its original state.

Let $f(x) = a(x - h)^2 + k$. The vertex of the graph of f is at $(2, 3)$ and the graph passes through $(1, 7)$.

a. Write down the value of h and of k. [2]

b. Find the value of a. [3]

Markscheme

a. $h = 2,\ k = 3$ ***A1A1 N2***

[2 marks]

b. attempt to substitute $(1, 7)$ in any order into their $f(x)$ ***(M1)***

eg $7 = a(1-2)^2 + 3, 7 = a(1-3)^2 + 2, 1 = a(7-2)^2 + 3$

correct equation ***(A1)***

eg $7 = a + 3$

$a = 4$ ***A1*** ***N2***

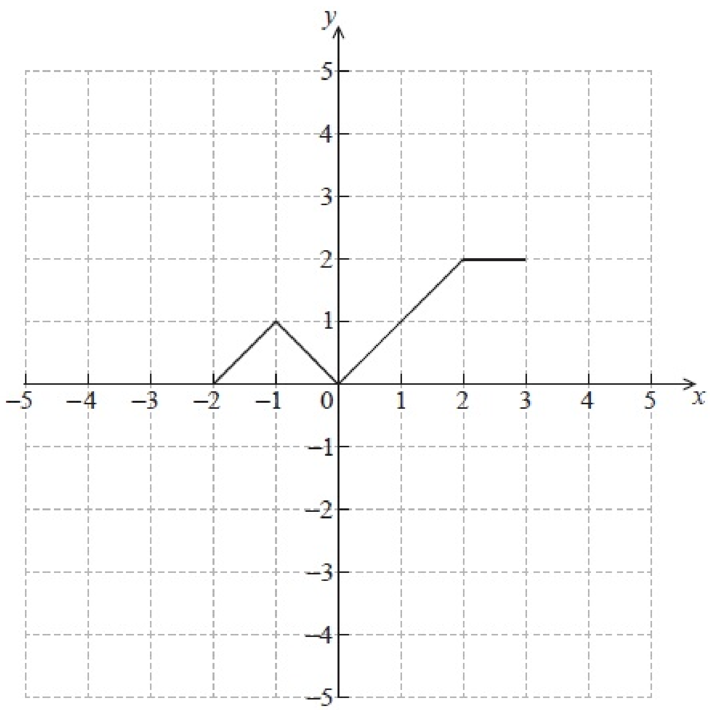
[3 marks]

Examiners report

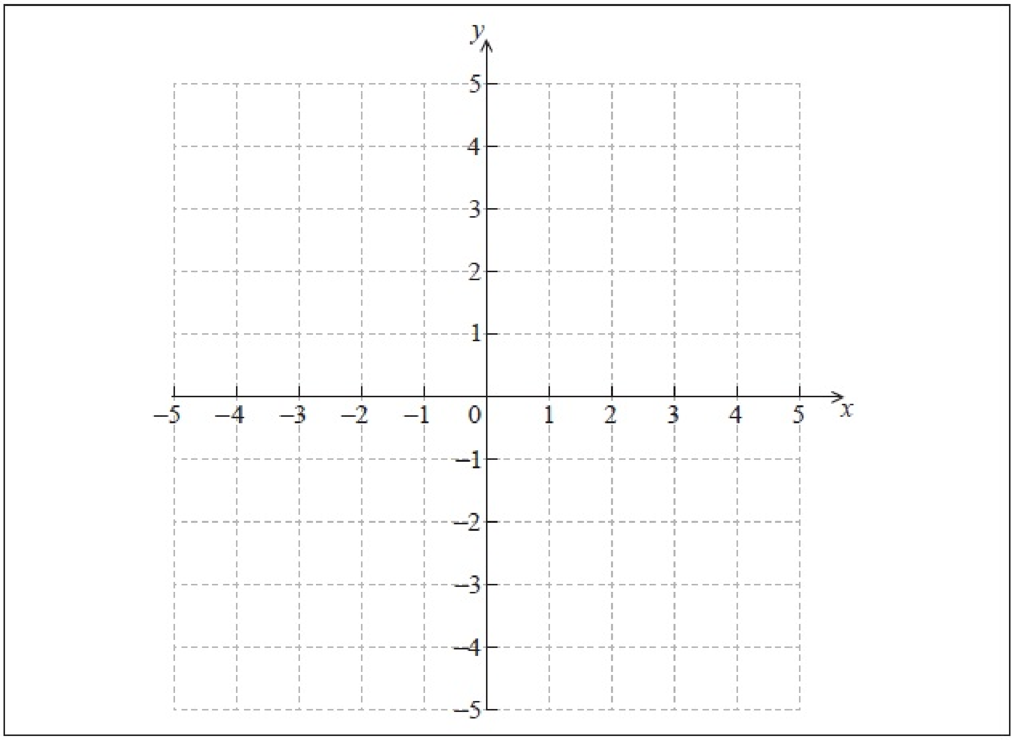
a. [N/A]

b. [N/A]

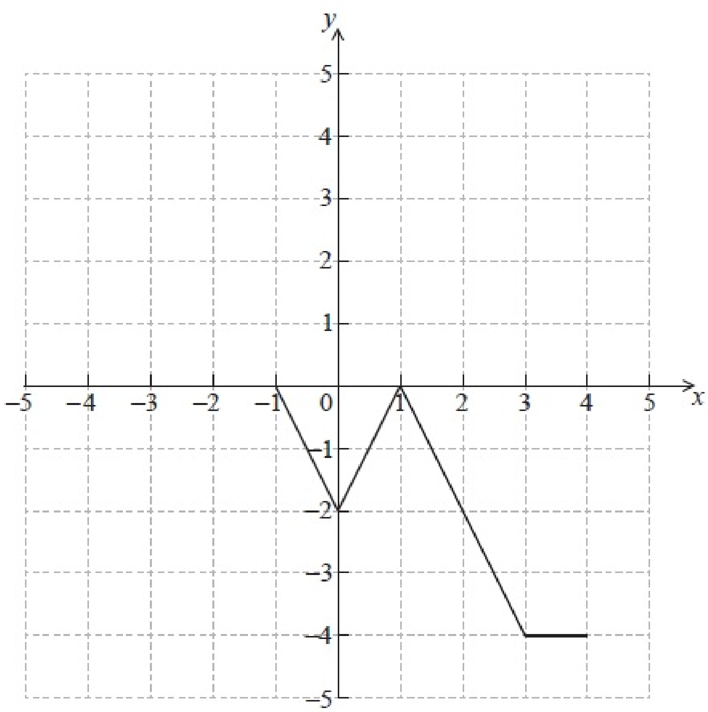
The diagram below shows the graph of a function $f(x)$, for $-2 \le x \le 3$.



a. Sketch the graph of $f(-x)$ on the grid below. [2]



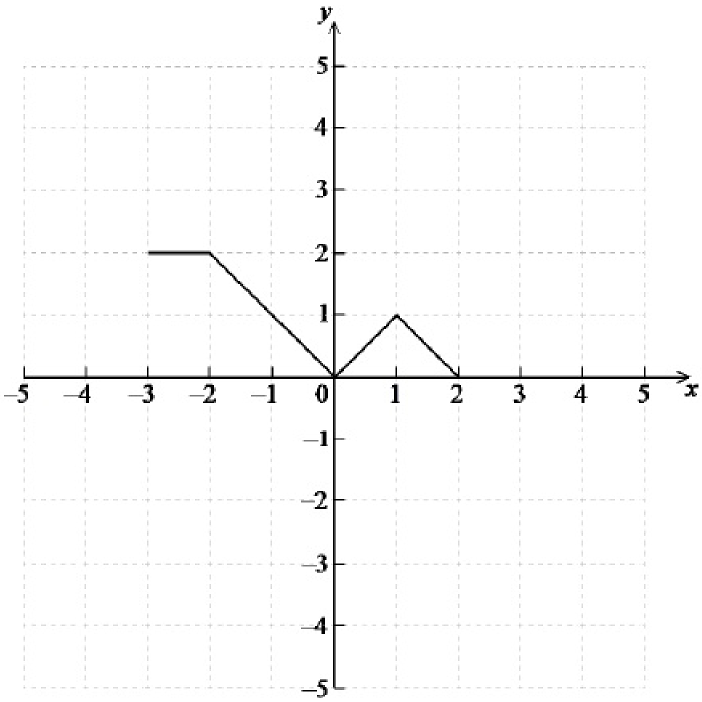
b. The graph of *f* is transformed to obtain the graph of g . The graph of g is shown below. [4]



The function g can be written in the form $g(x) = af(x + b)$. Write down the value of a and of b .

Markscheme

a.



A2 *N2*

[2 marks]

b. $a=-2, b=-1$ ***A2A2*** ***N4***

Note: Award ***A1*** for $a=2$, ***A1*** for $b=1$.

[4 marks]

Examiners report

a. In part (a) of this question, a large number of candidates correctly sketched the graph of $f(-x)$, as asked. A fairly common error, however, was to graph $-f(x)$.

b. In part (b), many candidates seemed to recognize that the value of *a* was related to a vertical stretch, though some omitted the negative required for the vertical reflection. Similarly, some candidates gave a positive value for *b*.

Let $f(x)=x^2+4$ and $g(x)=x-1$.

a. Find $(f\circ g)(x)$. [2]

b. The vector $\begin{pmatrix}3\\-1\end{pmatrix}$ translates the graph of $(f\circ g)$ to the graph of h . [3]

Find the coordinates of the vertex of the graph of h .

c. The vector $\begin{pmatrix}3\\-1\end{pmatrix}$ translates the graph of $(f\circ g)$ to the graph of h . [2]

Show that $h(x)=x^2-8x+19$.

d. The vector $\begin{pmatrix}3\\-1\end{pmatrix}$ translates the graph of $(f\circ g)$ to the graph of h . [5]

The line $y = 2x - 6$ is a tangent to the graph of h at the point P. Find the x-coordinate of P.

Markscheme

a. attempt to form composition (in any order) ***(M1)***

$(f \circ g)(x) = (x - 1)^2 + 4 \quad (x^2 - 2x + 5)$ ***A1 N2***

[2 marks]

b. **METHOD 1**

vertex of $f \circ g$ at (1, 4) ***(A1)***

evidence of appropriate approach ***(M1)***

e.g. adding $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ to the coordinates of the vertex of $f \circ g$

vertex of h at (4, 3) ***A1 N3***

METHOD 2

attempt to find $h(x)$ ***(M1)***

e.g. $((x - 3) - 1)^2 + 4 - 1$, $h(x) = (f \circ g)(x - 3) - 1$

$h(x) = (x - 4)^2 + 3$ ***(A1)***

vertex of h at (4, 3) ***A1 N3***

[3 marks]

c. evidence of appropriate approach ***(M1)***

e.g. $(x - 4)^2 + 3$,$(x - 3)^2 - 2(x - 3) + 5 - 1$

simplifying ***A1***

e.g. $h(x) = x^2 - 8x + 16 + 3$, $x^2 - 6x + 9 - 2x + 6 + 4$

$h(x) = x^2 - 8x + 19$ ***AG N0***

[2 marks]

d. **METHOD 1**

equating functions to find intersection point ***(M1)***

e.g. $x^2 - 8x + 19 = 2x - 6$, $y = h(x)$

$x^2 - 10x + 25 + 0$ ***A1***

evidence of appropriate approach to solve ***(M1)***

e.g. factorizing, quadratic formula

appropriate working ***A1***

e.g. $(x - 5)^2 = 0$

$x = 5 \ \ (p = 5)$ ***A1 N3***

METHOD 2

attempt to find $h'(x)$ ***(M1)***

$h(x) = 2x - 8$ ***A1***

recognizing that the gradient of the tangent is the derivative ***(M1)***

e.g. gradient at $p = 2$

$2x - 8 = 2 \ \ (2x = 10)$ ***A1***

$x = 5$ ***A1 N3***

[5 marks]

Examiners report

a. Candidates showed good understanding of finding the composite function in part (a).

b. There were some who did not seem to understand what the vector translation meant in part (b).

c. Candidates showed good understanding of manipulating the quadratic in part (c).

d. There was more than one method to solve for h in part (d), and a pleasing number of candidates were successful in this part of the question.

Let $f(x) = m - \frac{1}{x}$, for $x \neq 0$. The line $y = x - m$ intersects the graph of f in two distinct points. Find the possible values of m.

Markscheme

valid approach ***(M1)***

eg $f = y,\ m - \frac{1}{x} = x - m$

correct working to eliminate denominator ***(A1)***

eg $mx - 1 = x(x - m),\ mx - 1 = x^2 - mx$

correct quadratic equal to zero ***A1***

eg $x^2 - 2mx + 1 = 0$

correct reasoning ***R1***

eg for two solutions, $b^2 - 4ac > 0$

correct substitution into the discriminant formula ***(A1)***

eg $(-2m)^2 - 4$

correct working ***(A1)***

eg $4m^2 > 4,\ m^2 = 1$, sketch of positive parabola on the x-axis

correct interval ***A1*** ***N4***

eg $|m| > 1,\ m < -1$ or $m > 1$

[7 marks]

Examiners report

[N/A]

Let $f(x) = 2x^3 + 3$ and $g(x) = e^{3x} - 2$.

a. (i) Find $g(0)$. [5]

(ii) Find $(f \circ g)(0)$.

b. Find $f^{-1}(x)$. [3]

Markscheme

a. (i) $g(0) = \mathrm{e}^0 - 2$ ***(A1)***

$= -1$ ***A1*** ***N2***

(ii) **METHOD 1**

substituting answer from (i) ***(M1)***

e.g. $(f \circ g)(0) = f(-1)$

correct substitution $f(-1) = 2(-1)^3 + 3$ ***(A1)***

$f(-1) = 1$ ***A1*** ***N3***

METHOD 2

attempt to find $(f \circ g)(x)$ ***(M1)***

e.g. $(f \circ g)(x) = f(\mathrm{e}^{3x} - 2) = 2(\mathrm{e}^{3x} - 2)^3 + 3$

correct expression for $(f \circ g)(x)$ ***(A1)***

e.g. $2(\mathrm{e}^{3x} - 2)^3 + 3$

$(f \circ g)(0) = 1$ ***A1*** ***N3***

[5 marks]

b. interchanging *x* and *y* (seen anywhere) ***(M1)***

e.g. $x = 2y^3 + 3$

attempt to solve ***(M1)***

e.g. $y^3 = \frac{x-3}{2}$

$f^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$ ***A1*** ***N3***

[3 marks]

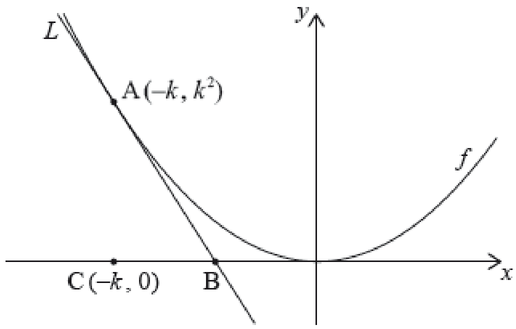
Examiners report

a. This question was generally done well, although some students consider e^0 to be 0, losing them a mark.

b. A few candidates composed in the wrong order. Most found the formula of the inverse correctly, even if in some cases there were errors when trying to isolate *x* (or *y*). A common incorrect solution found was to find $y = \sqrt[3]{\frac{x-3}{2}}$.

Let $f(x) = x^2$. The following diagram shows part of the graph of f.

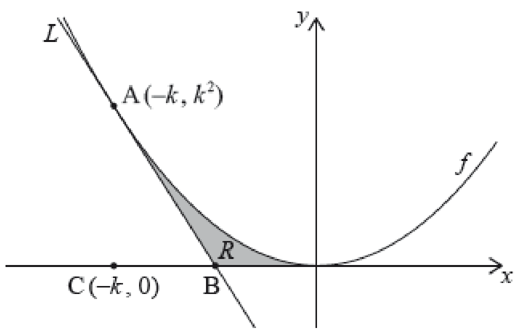
diagram not to scale



The line L is the tangent to the graph of f at the point $\mathrm{A}(-k,\ k^2)$, and intersects the x-axis at point B. The point C is $(-k,\ 0)$.

The region R is enclosed by L, the graph of f, and the x-axis. This is shown in the following diagram.

diagram not to scale



a.i. Write down $f'(x)$. [1]

a.ii. Find the gradient of L. [2]

b. Show that the x-coordinate of B is $-\frac{k}{2}$. [5]

c. Find the area of triangle ABC, giving your answer in terms of k. [2]

d. Given that the area of triangle ABC is p times the area of R, find the value of p. [7]

Markscheme

a.i. $f'(x) = 2x$ ***A1 N1***

[1 mark]

a.ii. attempt to substitute $x = -k$ into their derivative ***(M1)***

gradient of L is $-2k$ ***A1 N2***

[2 marks]

b. **METHOD 1**

attempt to substitute coordinates of A and their gradient into equation of a line ***(M1)***

eg $k^2 = -2k(-k) + b$

correct equation of L in any form ***(A1)***

eg $y - k^2 = -2k(x + k),\ y = -2kx - k^2$

valid approach ***(M1)***

eg $y = 0$

correct substitution into L equation ***A1***

eg $-k^2 = -2kx - 2k^2,\ 0 = -2kx - k^2$

correct working ***A1***

eg $2kx = -k^2$

$x = -\frac{k}{2}$ ***AG*** ***N0***

METHOD 2

valid approach ***(M1)***

eg $\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1},\ -2k = \frac{\text{rise}}{\text{run}}$

recognizing $y = 0$ at B ***(A1)***

attempt to substitute coordinates of A and B into slope formula ***(M1)***

eg $\frac{k^2-0}{-k-x},\ \frac{-k^2}{x+k}$

correct equation ***A1***

eg $\frac{k^2-0}{-k-x} = -2k,\ \frac{-k^2}{x+k} = -2k,\ -k^2 = -2k(x+k)$

correct working ***A1***

eg $2kx = -k^2$

$x = -\frac{k}{2}$ ***AG*** ***N0***

[5 marks]

c. valid approach to find area of triangle ***(M1)***

eg $\frac{1}{2}(k^2)\left(\frac{k}{2}\right)$

area of $\text{ABC} = \frac{k^3}{4}$ ***A1*** ***N2***

[2 marks]

d. **METHOD 1 ($\int f - \text{triangle}$)**

valid approach to find area from $-k$ to 0 ***(M1)***

eg $\int_{-k}^{0} x^2 \mathrm{d}x,\ \int_{0}^{-k} f$

correct integration (seen anywhere, even if ***M0*** awarded) ***A1***

eg $\frac{x^3}{3},\ \left[\frac{1}{3}x^3\right]_{-k}^{0}$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

eg $0 - \frac{(-k)^3}{3}$, area from $-k$ to 0 is $\frac{k^3}{3}$

Note: Award ***M0*** for substituting into original or differentiated function.

attempt to find area of R ***(M1)***

eg $\int_{-k}^{0} f(x)\mathrm{d}x -\ \text{triangle}$

correct working for R ***(A1)***

eg $\frac{k^3}{3} - \frac{k^3}{4},\ R = \frac{k^3}{12}$

correct substitution into $\text{triangle} = pR$ ***(A1)***

eg $\frac{k^3}{4} = p\left(\frac{k^3}{3} - \frac{k^3}{4}\right),\ \frac{k^3}{4} = p\left(\frac{k^3}{12}\right)$

$p=3$ ***A1*** ***N2***

METHOD 2 ($\int(f-L)$)

valid approach to find area of R ***(M1)***

eg $\int_{-k}^{-\frac{k}{2}} x^2-(-2kx-k^2)\mathrm{d}x+\int_{-\frac{k}{2}}^{0} x^2\mathrm{d}x,\ \int_{-k}^{-\frac{k}{2}}(f-L)+\int_{-\frac{k}{2}}^{0} f$

correct integration (seen anywhere, even if ***M0*** awarded) ***A2***

eg $\frac{x^3}{3}+kx^2+k^2x,\ \left[\frac{x^3}{3}+kx^2+k^2x\right]_{-k}^{-\frac{k}{2}}+\left[\frac{x^3}{3}\right]_{-\frac{k}{2}}^{0}$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

eg $\left(\frac{\left(-\frac{k}{2}\right)^3}{3}+k\left(-\frac{k}{2}\right)^2+k^2\left(-\frac{k}{2}\right)\right)-\left(\frac{(-k)^3}{3}+k(-k)^2+k^2(-k)\right)+(0)-\left(\frac{\left(-\frac{k}{2}\right)^3}{3}\right)$

Note: Award ***M0*** for substituting into original or differentiated function.

correct working for R ***(A1)***

eg $\frac{k^3}{24}+\frac{k^3}{24},\ -\frac{k^3}{24}+\frac{k^3}{4}-\frac{k^3}{2}+\frac{k^3}{3}-k^3+k^3+\frac{k^3}{24},\ R=\frac{k^3}{12}$

correct substitution into $\text{triangle}=pR$ ***(A1)***

eg $\frac{k^3}{4}=p\left(\frac{k^3}{24}+\frac{k^3}{24}\right),\ \frac{k^3}{4}=p\left(\frac{k^3}{12}\right)$

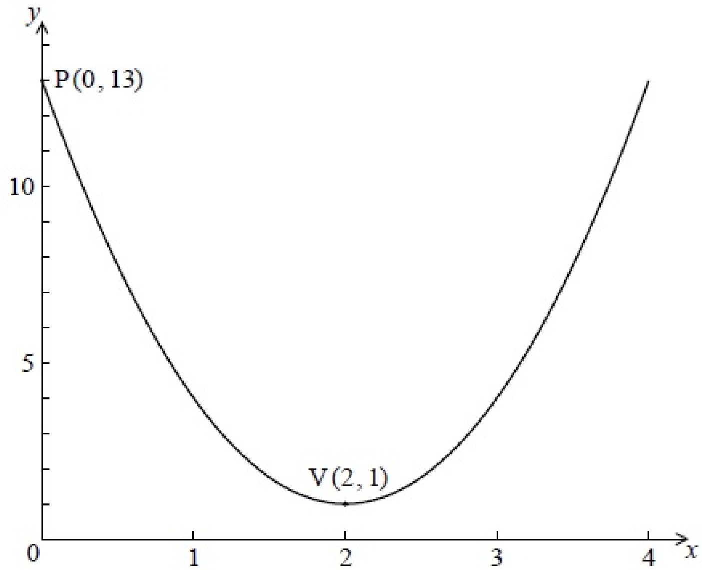
$p=3$ ***A1*** ***N2***

[7 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

The following diagram shows the graph of a quadratic function f, for $0\le x\le 4$.



The graph passes through the point P(0, 13) , and its vertex is the point V(2, 1) .

a(i) and (ii). The function can be written in the form $f(x) = a(x - h)^2 + k$. [4]

(i) Write down the value of h and of k .

(ii) Show that $a = 3$.

b. Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$. [3]

c. Calculate the area enclosed by the graph of f , the x-axis, and the lines $x = 2$ and $x = 4$. [8]

Markscheme

a(i) and (ii). (i) $h = 2$, $k = 1$ ***A1A1*** ***N2***

(ii) attempt to substitute coordinates of any point (except the vertex) on the graph into f ***M1***

e.g. $13 = a(0 - 2)^2 + 1$

working towards solution ***A1***

e.g. $13 = 4a + 1$

$a = 3$ ***AG*** ***N0***

[4 marks]

b. attempting to expand **their** binomial ***(M1)***

e.g. $f(x) = 3(x^2 - 2 \times 2x + 4) + 1$, $(x - 2)^2 = x^2 - 4x + 4$

correct working ***(A1)***

e.g. $f(x) = 3x^2 - 12x + 12 + 1$

$f(x) = 3x^2 - 12x + 13$ (accept $A = 3$, $B = -12$, $C = 13$) ***A1*** ***N2***

[3 marks]

c. **METHOD 1**

integral expression ***(A1)***

e.g. $\int_2^4 (3x^2 - 12x + 13)$, $\int f \mathrm{d}x$

Area $= [x^3 - 6x^2 + 13x]_2^4$ ***A1A1A1***

Note: Award ***A1*** for x^3 , ***A1*** for $-6x^2$, ***A1*** for $13x$.

correct substitution of **correct** limits into **their** expression ***A1A1***

e.g. $(4^3 - 6 \times 4^2 + 13 \times 4) - (2^3 - 6 \times 2^2 + 13 \times 2)$, $64 - 96 + 52 - (8 - 24 + 26)$

Note: Award ***A1*** for substituting 4, ***A1*** for substituting 2.

correct working ***(A1)***

e.g. $64 - 96 + 52 - 8 + 24 - 26, 20 - 10$

Area $= 10$ ***A1*** ***N3***

[8 marks]

METHOD 2

integral expression ***(A1)***

e.g. $\int_2^4 (3(x-2)^2 + 1)$, $\int f \mathrm{d}x$

Area $= [(x-2)^3 + x]_2^4$ ***A2A1***

Note: Award ***A2*** for $(x-2)^3$, ***A1*** for x .

correct substitution of **correct** limits into **their** expression ***A1A1***

e.g. $(4-2)^3 + 4 - [(2-2)^3 + 2]$, $2^3 + 4 - (0^3 + 2)$, $2^3 + 4 - 2$

Note: Award ***A1*** for substituting 4, ***A1*** for substituting 2.

correct working ***(A1)***

e.g. $8 + 4 - 2$

Area $= 10$ ***A1*** ***N3***

[8 marks]

METHOD 3

recognizing area from 0 to 2 is same as area from 2 to 4 ***(R1)***

e.g. sketch, $\int_2^4 f = \int_0^2 f$

integral expression ***(A1)***

e.g. $\int_0^2 (3x^2 - 12x + 13)$, $\int f \mathrm{d}x$

Area $= [x^3 - 6x^2 + 13x]_0^2$ ***A1A1A1***

Note: Award ***A1*** for x^3 , ***A1*** for $-6x^2$, ***A1*** for $13x$.

correct substitution of **correct** limits into **their** expression ***A1(A1)***

e.g. $(2^3 - 6 \times 2^2 + 13 \times 2) - (0^3 - 6 \times 0^2 + 13 \times 0)$, $8 - 24 + 26$

Note: Award ***A1*** for substituting 2, ***(A1)*** for substituting 0.

Area $= 10$ ***A1*** ***N3***

[8 marks]

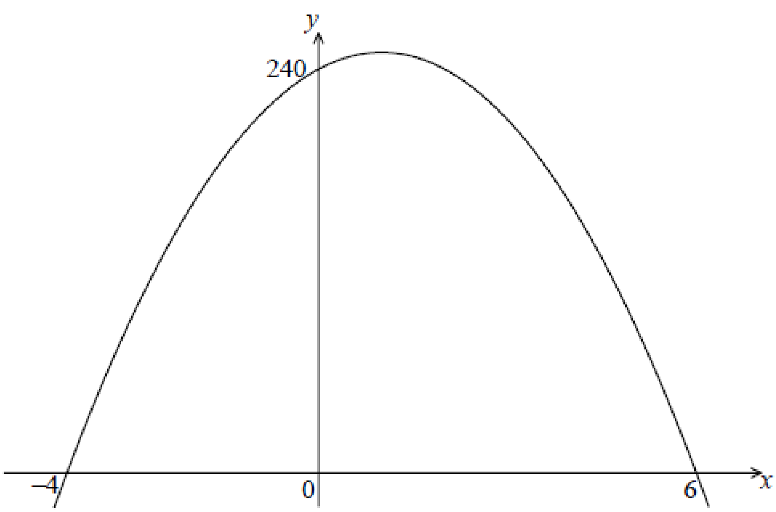
Examiners report

a(i) and (ii). In part (a), nearly all the candidates recognized that h and k were the coordinates of the vertex of the parabola, and most were able to successfully show that $a = 3$. Unfortunately, a few candidates did not understand the "show that" command, and simply verified that $a = 3$ would work, rather than showing how to find $a = 3$.

b. In part (b), most candidates were able to find $f(x)$ in the required form. For a few candidates, algebraic errors kept them from finding the correct function, even though they started with correct values for a, h and k.

c. In part (c), nearly all candidates knew that they needed to integrate to find the area, but errors in integration, and algebraic and arithmetic errors prevented many from finding the correct area.

The following diagram shows part of the graph of a quadratic function f .



The x-intercepts are at $(-4, 0)$ and $(6, 0)$, and the y-intercept is at $(0, 240)$.

a. Write down $f(x)$ in the form $f(x) = -10(x-p)(x-q)$. [2]

b. Find another expression for $f(x)$ in the form $f(x) = -10(x-h)^2 + k$. [4]

c. Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$. [2]

d(i) and (ii). A particle moves along a straight line so that its velocity, v ms^{-1} , at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \le t \le 6$. [7]

(i) Find the value of t when the speed of the particle is greatest.

(ii) Find the acceleration of the particle when its speed is zero.

Markscheme

a. $f(x) = -10(x+4)(x-6)$ ***A1A1*** ***N2***

[2 marks]

b. **METHOD 1**

attempting to find the x-coordinate of maximum point ***(M1)***

e.g. averaging the x-intercepts, sketch, $y' = 0$, axis of symmetry

attempting to find the y-coordinate of maximum point ***(M1)***

e.g. $k = -10(1+4)(1-6)$

$f(x) = -10(x-1)^2 + 250$ ***A1A1*** ***N4***

METHOD 2

attempt to expand $f(x)$ ***(M1)***

e.g. $-10(x^2 - 2x - 24)$

attempt to complete the square ***(M1)***

e.g. $-10((x-1)^2 - 1 - 24)$

$f(x) = -10(x-1)^2 + 250$ ***A1A1*** ***N4***

[4 marks]

c. attempt to simplify ***(M1)***

e.g. distributive property, $-10(x-1)(x-1) + 250$

correct simplification ***A1***

e.g. $-10(x^2 - 6x + 4x - 24)$, $-10(x^2 - 2x + 1) + 250$

$f(x) = 240 + 20x - 10x^2$ ***AG*** ***N0***

[2 marks]

d(i) and (ii). (i) valid approach ***(M1)***

e.g. vertex of parabola, $v'(t) = 0$

$t = 1$ ***A1*** ***N2***

(ii) recognizing $a(t) = v'(t)$ ***(M1)***

$a(t) = 20 - 20t$ ***A1A1***

speed is zero $\Rightarrow t = 6$ ***(A1)***

$a(6) = -100$ (ms^{-2}) ***A1*** ***N3***

[7 marks]

Examiners report

a. Parts (a) and (c) of this question were very well done by most candidates.

b. In part (b), many candidates attempted to use the method of completing the square, but were unsuccessful dealing with the coefficient of -10. Candidates who recognized that the x-coordinate of the vertex was 1, then substituted this value into the function from part (a), were generally able to earn full marks here.

c. Parts (a) and (c) of this question were very well done by most candidates.

d(i) and (ii). In part (d), it was clear that many candidates were not familiar with the relationship between velocity and acceleration, and did not understand how those concepts were related to the graph which was given. A large number of candidates used time $t = 1$ in part b(ii), rather than $t = 6$. To find the acceleration, some candidates tried to integrate the velocity function, rather than taking the derivative of velocity. Still others found the derivative in part b(i), but did not realize they needed to use it in part b(ii), as well.

Let $f(x) = x^2 - 4x + 5$.

The function can also be expressed in the form $f(x) = (x - h)^2 + k$.

a. Find the equation of the axis of symmetry of the graph of f. [2]

b. (i) Write down the value of h. [4]

(ii) Find the value of k.

Markscheme

a. correct approach ***(A1)***

eg $\frac{-(-4)}{2},\ f'(x) = 2x - 4 = 0,\ (x^2 - 4x + 4) + 5 - 4$

$x = 2$ (must be an equation) ***A1 N2***

[2 marks]

b. (i) $h = 2$ ***A1 N1***

(ii) **METHOD 1**

valid attempt to find k ***(M1)***

eg $f(2)$

correct substitution into **their** function ***(A1)***

eg $(2)^2 - 4(2) + 5$

$k = 1$ ***A1 N2***

METHOD 2

valid attempt to complete the square ***(M1)***

eg $x^2 - 4x + 4$

correct working ***(A1)***

eg $(x^2 - 4x + 4) - 4 + 5,\ (x - 2)^2 + 1$

$k = 1$ ***A1 N2***

[4 marks]

Examiners report

a. [N/A]

b. [N/A]

Let $f(x) = \cos 2x$ and $g(x) = 2x^2 - 1$.

a. Find $f\left(\frac{\pi}{2}\right)$. [2]

b. Find $(g \circ f)\left(\frac{\pi}{2}\right)$. [2]

c. Given that $(g \circ f)(x)$ can be written as $\cos(kx)$, find the value of k, $k \in \mathbb{Z}$. [3]

Markscheme

a. $f\left(\frac{\pi}{2}\right) = \cos \pi$ ***(A1)***

$= -1$ ***A1 N2***

[2 marks]

b. $(g \circ f)\left(\frac{\pi}{2}\right) = g(-1)\ (= 2(-1)^2 - 1)$ ***(A1)***

$= 1$ ***A1 N2***

[2 marks]

c. $(g \circ f)(x) = 2(\cos(2x))^2 - 1\ (= 2\cos^2(2x) - 1)$ ***A1***

evidence of $2\cos^2\theta - 1 = \cos 2\theta$ (seen anywhere) ***(M1)***

$(g \circ f)(x) = \cos 4x$

$k = 4$ ***A1 N2***

[3 marks]

Examiners report

a. In part (a), a number of candidates were not able to evaluate $\cos \pi$, either leaving it or evaluating it incorrectly.

b. Almost all candidates evaluated the composite function in part (b) in the given order, many earning follow-through marks for incorrect answers from part (a). On both parts (a) and (b), there were candidates who correctly used double-angle formulas to come up with correct answers; while this is a valid method, it required unnecessary additional work.

c. Candidates were not as successful in part (c). Many tried to use double-angle formulas, but either used the formula incorrectly or used it to write the expression in terms of $\cos x$ and went no further. There were a number of cases in which the candidates "accidentally" came up with the correct answer based on errors or lucky guesses and did not earn credit for their final answer. Only a few candidates recognized the correct method of solution.

Let $f(x) = \ln(x + 5) + \ln 2$, for $x > -5$.

a. Find $f^{-1}(x)$. [4]

b. Let $g(x) = \mathrm{e}^x$. [3]

Find $(g \circ f)(x)$, giving your answer in the form $ax + b$, where $a, b \in \mathbb{Z}$.

Markscheme

a. **METHOD 1**

$\ln(x+5) + \ln 2 = \ln(2(x+5))\ (= \ln(2x+10))$ ***(A1)***

interchanging x and y (seen anywhere) ***(M1)***

e.g. $x = \ln(2y+10)$

evidence of correct manipulation ***(A1)***

e.g. $\mathrm{e}^x = 2y + 10$

$f^{-1}(x) = \frac{\mathrm{e}^x - 10}{2}$ ***A1*** ***N2***

METHOD 2

$y = \ln(x+5) + \ln 2$

$y - \ln 2 = ln(x+5)$ ***(A1)***

evidence of correct manipulation ***(A1)***

e.g. $\mathrm{e}^{y-\ln 2} = x + 5$

interchanging x and y (seen anywhere) ***(M1)***

e.g. $\mathrm{e}^{x-\ln 2} = y + 5$

$f^{-1}(x) = \mathrm{e}^{x-\ln 2} - 5$ ***A1*** ***N2***

[4 marks]

b. **METHOD 1**

evidence of composition in correct order ***(M1)***

e.g. $(g \circ f)(x) = g(\ln(x+5) + \ln 2)$

$= \mathrm{e}^{\ln(2(x+5))} = 2(x+5)$

$(g \circ f)(x) = 2x + 10$ ***A1A1*** ***N2***

METHOD 2

evidence of composition in correct order ***(M1)***

e.g. $(g \circ f)(x) = \mathrm{e}^{\ln(x+5)+\ln 2}$

$= \mathrm{e}^{\ln(x+5)} \times \mathrm{e}^{\ln 2} = (x+5)2$

$(g \circ f)(x) = 2x + 10$ ***A1A1*** ***N2***

[3 marks]

Examiners report

a. This was one of the more difficult problems for the candidates. Knowledge of the laws of logarithms appeared weak as did the inverse nature of the exponential and logarithmic functions. There were a number of candidates who mistook the notation for the inverse to mean either the derivative or the reciprocal. The order of composition seemed well understood by most candidates but they were unable to simplify by the rules of indices to obtain the correct final answer.

b. This was one of the more difficult problems for the candidates. Knowledge of the laws of logarithms appeared weak as did the inverse nature of the exponential and logarithmic functions. There were a number of candidates who mistook the notation for the inverse to mean either the derivative or the reciprocal. The order of composition seemed well understood by most candidates but they were unable to simplify by the rules of indices to obtain the correct final answer.

Let $f(x) = k\log_2 x$.

a. Given that $f^{-1}(1) = 8$, find the value of k . [3]

b. Find $f^{-1}\left(\frac{2}{3}\right)$. [4]

Markscheme

a. **METHOD 1**

recognizing that $f(8) = 1$ ***(M1)***

e.g. $1 = k\log_2 8$

recognizing that $\log_2 8 = 3$ ***(A1)***

e.g. $1 = 3k$

$k = \frac{1}{3}$ ***A1*** ***N2***

METHOD 2

attempt to find the inverse of $f(x) = k\log_2 x$ ***(M1)***

e.g. $x = k\log_2 y$, $y = 2^{\frac{x}{k}}$

substituting 1 and 8 ***(M1)***

e.g. $1 = k\log_2 8$, $2^{\frac{1}{k}} = 8$

$k = \frac{1}{\log_2 8}$ $\left(k = \frac{1}{3}\right)$ ***A1*** ***N2***

[3 marks]

b. **METHOD 1**

recognizing that $f(x) = \frac{2}{3}$ ***(M1)***

e.g. $\frac{2}{3} = \frac{1}{3}\log_2 x$

$\log_2 x = 2$ ***(A1)***

$f^{-1}\left(\frac{2}{3}\right) = 4$ (accept $x = 4$) ***A2*** ***N3***

METHOD 2

attempt to find inverse of $f(x) = \frac{1}{3}\log_2 x$ ***(M1)***

e.g. interchanging *x* and *y* , substituting $k = \frac{1}{3}$ into $y = 2^{\frac{x}{k}}$

correct inverse ***(A1)***

e.g. $f^{-1}(x) = 2^{3x}$, 2^{3x}

$f^{-1}\left(\frac{2}{3}\right) = 4$ ***A2*** ***N3***

[4 marks]

Examiners report

a. A very poorly done question. Most candidates attempted to find the inverse function for f and used that to answer parts (a) and (b). Few recognized that the explicit inverse function was not necessary to answer the question.

Although many candidates seem to know that they can find an inverse function by interchanging x and y, very few were able to actually get the correct inverse. Almost none recognized that if $f^{-1}(1) = 8$, then $f(8) = 1$. Many thought that the letters "log" could be simply "cancelled out", leaving the 2 and the 8.

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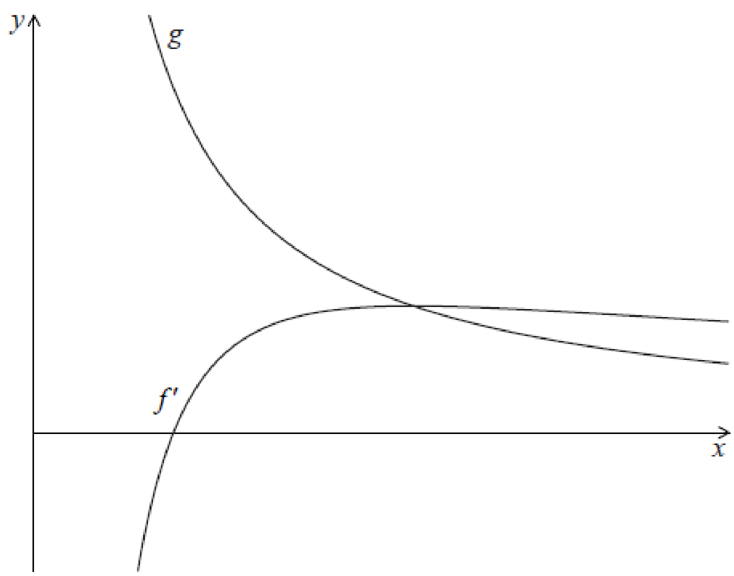
Let $f(x) = \frac{(\ln x)^2}{2}$, for $x > 0$.

Let $g(x) = \frac{1}{x}$. The following diagram shows parts of the graphs of f' and g.



The graph of f' has an x-intercept at $x = p$.

a. Show that $f'(x) = \frac{\ln x}{x}$. [2]

b. There is a minimum on the graph of f. Find the x-coordinate of this minimum. [3]

c. Write down the value of p. [2]

d. The graph of g intersects the graph of f' when $x = q$. [3]

Find the value of q.

e. The graph of g intersects the graph of f' when $x = q$. [5]

Let R be the region enclosed by the graph of f', the graph of g and the line $x = p$.
Show that the area of R is $\frac{1}{2}$.

Markscheme

a. **METHOD 1**

correct use of chain rule ***A1A1***

eg $\frac{2\ln x}{2}\times\frac{1}{x},\ \frac{2\ln x}{2x}$

Note: Award ***A1*** for $\frac{2\ln x}{2x}$, ***A1*** for $\times\frac{1}{x}$.

$f'(x)=\frac{\ln x}{x}$ ***AG*** ***N0***

[2 marks]

METHOD 2

correct substitution into quotient rule, with derivatives seen ***A1***

eg $\frac{2\times2\ln x\times\frac{1}{x}-0\times(\ln x)^2}{4}$

correct working ***A1***

eg $\frac{4\ln x\times\frac{1}{x}}{4}$

$f'(x)=\frac{\ln x}{x}$ ***AG*** ***N0***

[2 marks]

b. setting derivative $=0$ ***(M1)***

eg $f'(x)=0,\ \frac{\ln x}{x}=0$

correct working ***(A1)***

eg $\ln x=0,\ x=\mathrm{e}^0$

$x=1$ ***A1*** ***N2***

[3 marks]

c. intercept when $f'(x)=0$ ***(M1)***

$p=1$ ***A1*** ***N2***

[2 marks]

d. equating functions ***(M1)***

eg $f'=g,\ \frac{\ln x}{x}=\frac{1}{x}$

correct working ***(A1)***

eg $\ln x=1$

$q=\mathrm{e}$ (accept $x=\mathrm{e}$) ***A1*** ***N2***

[3 marks]

e. evidence of integrating and subtracting functions (in any order, seen anywhere) ***(M1)***

eg $\int_q^e\left(\frac{1}{x}-\frac{\ln x}{x}\right)\mathrm{d}x,\ \int f'-g$

correct integration $\ln x-\frac{(\ln x)^2}{2}$ ***A2***

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $(\ln\mathrm{e}-\ln 1)-\left(\frac{(\ln\mathrm{e})^2}{2}-\frac{(\ln 1)^2}{2}\right)$

Note: Do not award ***M1*** if the integrated function has only one term.

correct working ***A1***

eg $(1-0)-\left(\frac{1}{2}-0\right),\ 1-\frac{1}{2}$

area $=\frac{1}{2}$ ***AG*** ***N0***

Notes: Candidates may work with two separate integrals, and only combine them at the end. Award marks in line with the markscheme.

[5 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

Let $f(x) = 1 + \mathrm{e}^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.

a. Find $(g \circ f)(x)$. [2]

b. Given that $\lim\limits_{x \to +\infty} (g \circ f)(x) = -3$, find the value of b. [4]

Markscheme

a. attempt to form composite ***(M1)***

eg $g(1 + \mathrm{e}^{-x})$

correct function ***A1 N2***

eg $(g \circ f)(x) = 2 + b + 2\mathrm{e}^{-x},\ 2(1 + \mathrm{e}^{-x}) + b$

[2 marks]

b. evidence of $\lim\limits_{x \to \infty} (2 + b + 2\mathrm{e}^{-x}) = 2 + b + \lim\limits_{x \to \infty} (2\mathrm{e}^{-x})$ ***(M1)***

eg $2 + b + 2\mathrm{e}^{-\infty}$, graph with horizontal asymptote when $x \to \infty$

Note: Award ***M0*** if candidate clearly has incorrect limit, such as $x \to 0,\ \mathrm{e}^{\infty},\ 2\mathrm{e}^{0}$.

evidence that $\mathrm{e}^{-x} \to 0$ (seen anywhere) ***(A1)***

eg $\lim\limits_{x \to \infty} (\mathrm{e}^{-x}) = 0,\ 1 + \mathrm{e}^{-x} \to 1,\ 2(1) + b = -3,\ \mathrm{e}^{\text{large negative number}} \to 0$, graph of $y = \mathrm{e}^{-x}$ or

$y = 2\mathrm{e}^{-x}$ with asymptote $y = 0$, graph of composite function with asymptote $y = -3$

correct working ***(A1)***

eg $2 + b = -3$

$b = -5$ ***A1 N2***

[4 marks]

Examiners report

a. [N/A]
b. [N/A]

Let $f(x) = log_3 \sqrt{x}$, for $x > 0$.

a. Show that $f^{-1}(x) = 3^{2x}$. [2]

b. Write down the range of f^{-1} . [1]

c. Let $g(x) = \log_3 x$, for $x > 0$. [4]

Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

Markscheme

a. interchanging *x* and *y* (seen anywhere) ***(M1)***

e.g. $x = \log \sqrt{y}$ (accept any base)

evidence of correct manipulation ***A1***

e.g. $3^x = \sqrt{y}$, $3^y = x^{\frac{1}{2}}$, $x = \frac{1}{2}\log_3 y$, $2y = \log_3 x$

$f^{-1}(x) = 3^{2x}$ ***AG N0***

[2 marks]

b. $y > 0$, $f^{-1}(x) > 0$ ***A1 N1***

[1 mark]

c. **METHOD 1**

finding $g(2) = log_3 2$ (seen anywhere) ***A1***

attempt to substitute ***(M1)***

e.g. $(f^{-1} \circ g)(2) = 3^{2\log_3 2}$

evidence of using log or index rule ***(A1)***

e.g. $(f^{-1} \circ g)(2) = 3^{\log_3 4}$, $3^{\log_3 2^2}$

$(f^{-1} \circ g)(2) = 4$ ***A1 N1***

METHOD 2

attempt to form composite (in any order) ***(M1)***

e.g. $(f^{-1} \circ g)(x) = 3^{2\log_3 x}$

evidence of using log or index rule ***(A1)***

e.g. $(f^{-1} \circ g)(x) = 3^{\log_3 x^2}$, $3^{\log_3 x^2}$

$(f^{-1} \circ g)(x) = x^2$ ***A1***

$(f^{-1} \circ g)(2) = 4$ ***A1 N1***

[4 marks]

Examiners report

a. Candidates were generally skilled at finding the inverse of a logarithmic function.

b. Few correctly gave the range of this function, often stating “all real numbers” or “ $y \geq 0$ ”, missing the idea that the range of an inverse is the domain of the original function.

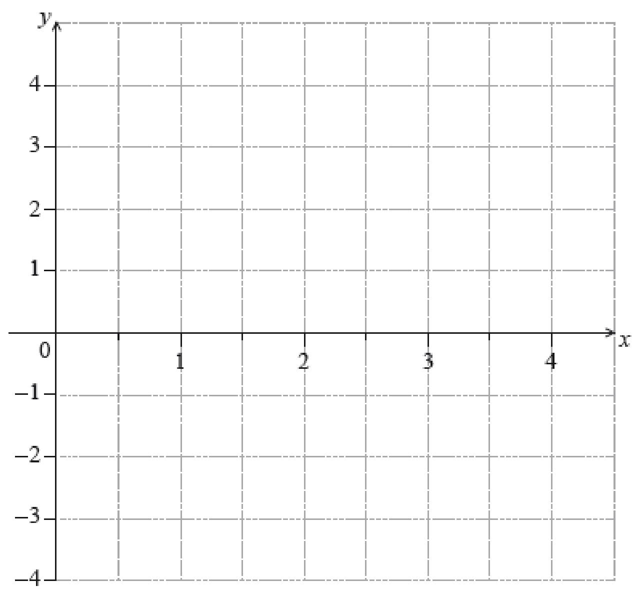
c. Some candidates answered part (c) correctly, although many did not get beyond $3^{2\log_3 2}$. Some attempted to form the composite in the incorrect order. Others interpreted $(f^{-1} \circ g)(2)$ as multiplication by 2.

Let $f(x) = 3\sin\left(\frac{\pi}{2}x\right)$, for $0 \leqslant x \leqslant 4$.

a. (i) Write down the amplitude of f. [3]

(ii) Find the period of f.

b. On the following grid sketch the graph of f. [4]



Markscheme

a. (i) 3 ***A1 N1***

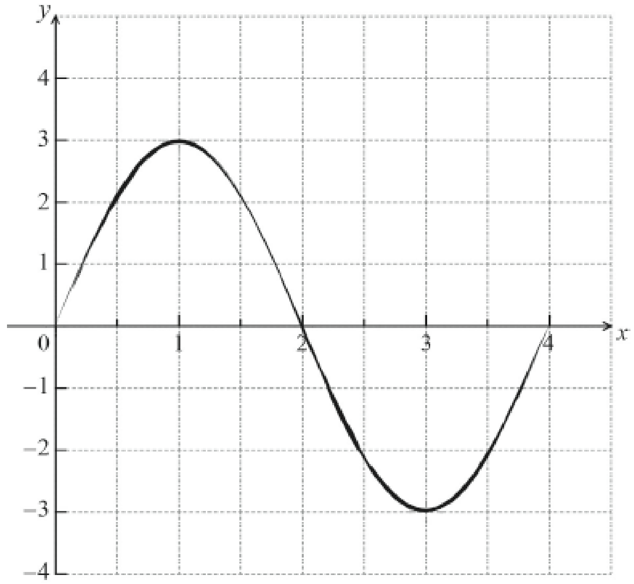
(ii) valid attempt to find the period ***(M1)***

eg $\frac{2\pi}{b}$, $\frac{2\pi}{\frac{\pi}{2}}$

period $= 4$ ***A1 N2***

[3 marks]

b.



A1A1A1A1 *N4*

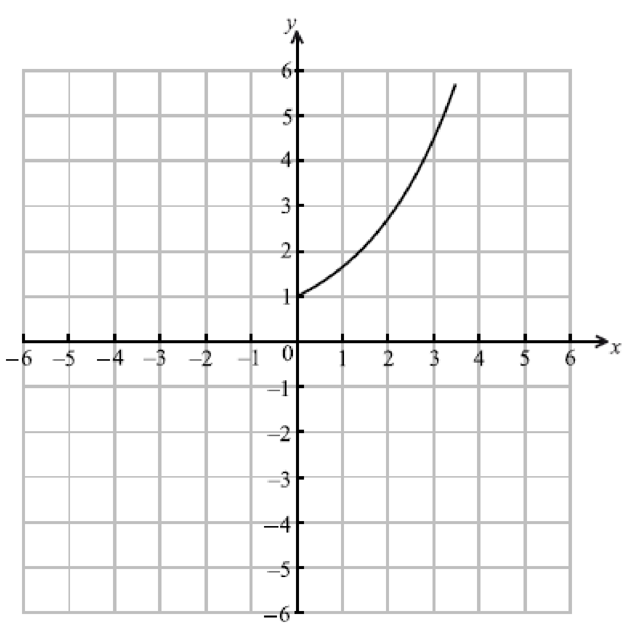
[4 marks]

Examiners report

a. Almost all candidates correctly stated the amplitude but then had difficulty finding the correct period. Few students faced problems in sketching the graph of the given function, even if they had found the wrong period, thus indicating a lack of understanding of the term 'period' in part a(ii). Most sketches were good although care should be taken to observe the given domain and to draw a neat curve.

b. Almost all candidates correctly stated the amplitude but then had difficulty finding the correct period. Few students faced problems in sketching the graph of the given function, even if they had found the wrong period, thus indicating a lack of understanding of the term 'period' in part a(ii). Most sketches were good although care should be taken to observe the given domain and to draw a neat curve.

Let *f* be the function given by $f(x) = \mathrm{e}^{0.5x}$, $0 \leq x \leq 3.5$. The diagram shows the graph of *f* .



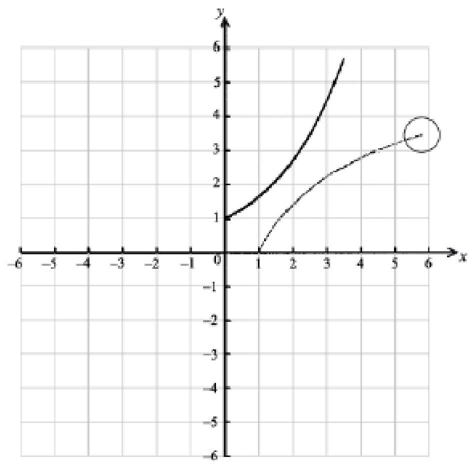
a. On the same diagram, sketch the graph of f^{-1} . [3]

b. Write down the range of f^{-1} . [1]

c. Find $f^{-1}(x)$. [3]

Markscheme

a.



A1A1A1 ***N3***

Note: Award ***A1*** for approximately correct (reflected) shape, ***A1*** for right end point in circle, ***A1*** for through $(1, 0)$.

b. $0 \le y \le 3.5$ ***A1*** ***N1***

[1 mark]

c. interchanging *x* and *y* (seen anywhere) ***M1***

e.g. $x = e^{0.5y}$

evidence of changing to log form ***A1***

e.g. $\ln x = 0.5y$, $\ln x = \ln \mathrm{e}^{0.5y}$ (any base), $\ln x = 0.5y \ln \mathrm{e}$ (any base)

$f^{-1}(x) = 2 \ln x$ ***A1*** ***N1***

[3 marks]

Examiners report

a. There were a large number of candidates who were unaware of the geometric relationship between a function and its inverse. Those that had some idea of the shape of the graph often did not consider the specified domain. Many more students were able to use an analytical approach to finding the inverse of a function and had little problem using logarithms to solve for *y*. Candidates were clearly more comfortable with algebraic procedures than graphical interpretations.

b. There were a large number of candidates who were unaware of the geometric relationship between a function and its inverse. Those that had some idea of the shape of the graph often did not consider the specified domain. Many more students were able to use an analytical approach to finding the inverse of a function and had little problem using logarithms to solve for *y*. Candidates were clearly more comfortable with algebraic procedures than graphical interpretations.

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Let $f(x) = 5x$ and $g(x) = x^2 + 1$, for $x \in \mathbb{R}$.

a. Find $f^{-1}(x)$. [2]

b. Find $(f \circ g)(7)$. [3]

Markscheme

a. interchanging x and x ***(M1)***

eg $x = 5y$

$f^{-1}(x) = \frac{x}{5}$ ***A1*** ***N2***

[2 marks]

b. **METHOD 1**

attempt to substitute 7 into $g(x)$ or $f(x)$ ***(M1)***

eg $7^2 + 1,\ 5 \times 7$

$g(7) = 50$ ***(A1)***

$f(50) = 250$ ***A1*** ***N2***

METHOD 2

attempt to form composite function (in any order) ***(M1)***

eg $5(x^2 + 1),\ (5x)^2 + 1$

correct substitution ***(A1)***

eg $5 \times (7^2 + 1)$

$(f \circ g)(7) = 250$ ***A1*** ***N2***

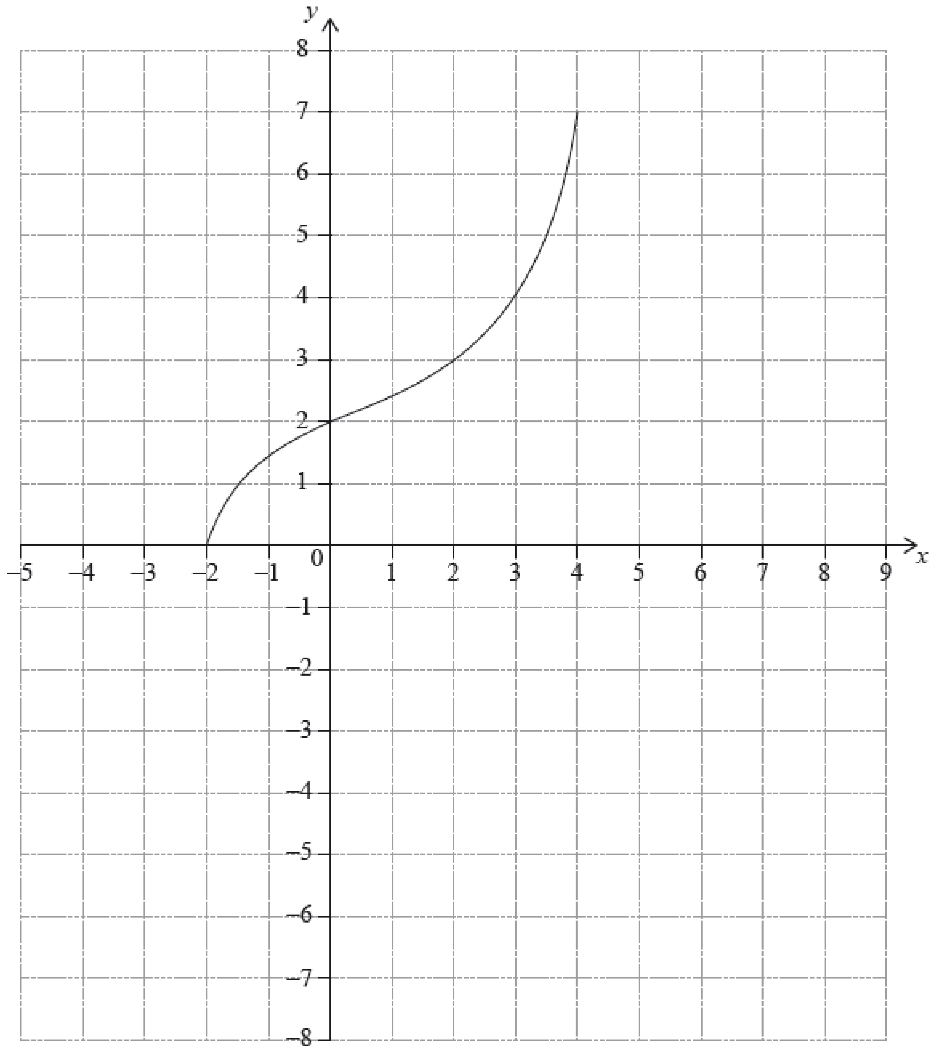
[3 marks]

Examiners report

a. [N/A]

b. [N/A]

The following diagram shows the graph of a function f, with domain $-2 \leqslant x \leqslant 4$.



The points $(-2,\ 0)$ and $(4,\ 7)$ lie on the graph of f.

a. Write down the range of f. [1]

b.i. Write down $f(2)$; [1]

b.ii. Write down $f^{-1}(2)$. [1]

c. On the grid, sketch the graph of f^{-1}. [3]

Markscheme

a. correct range (do not accept $0 \leqslant x \leqslant 7$) ***A1 N1***

eg $[0,\ 7],\ 0 \leqslant y \leqslant 7$

[1 mark]

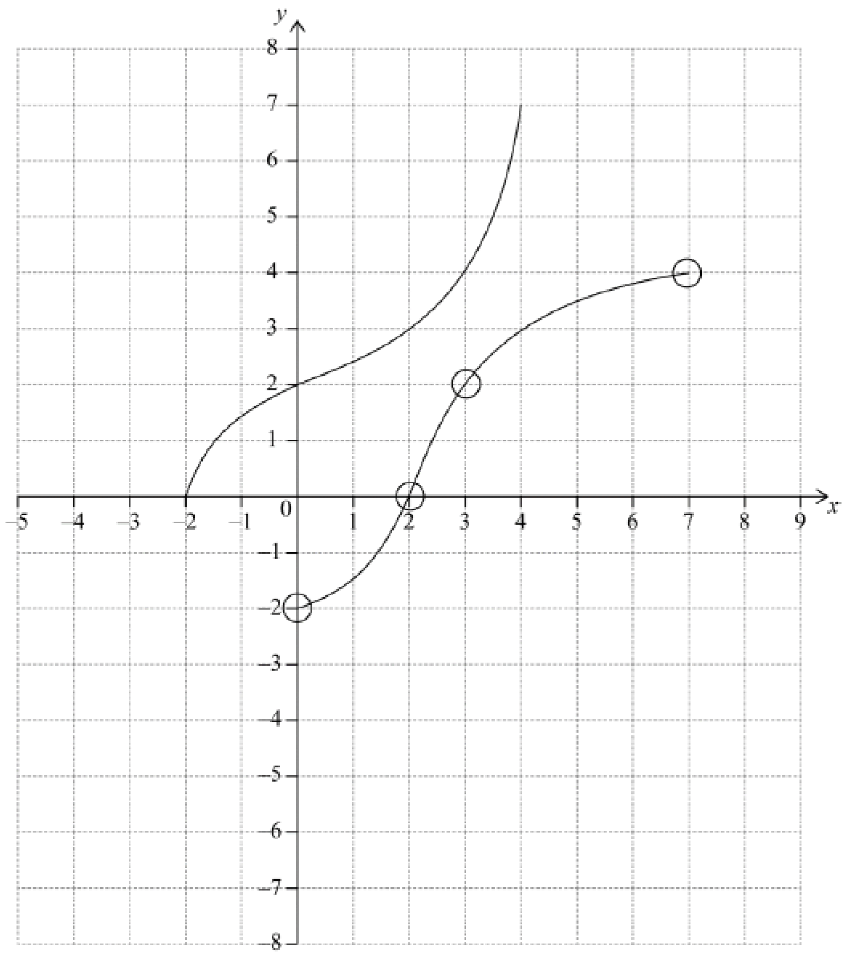
b.i. $f(2) = 3$ ***A1 N1***

[1 mark]

b.ii. $f^{-1}(2) = 0$ ***A1 N1***

[1 mark]

c.



A1A1A1 ***N3***

Notes: Award ***A1*** for both end points within circles,

A1 for images of $(2,\ 3)$ and $(0,\ 2)$ within circles,

A1 for approximately correct reflection in $y = x$, concave up then concave down shape (do not accept line segments).

[3 marks]

Examiners report

a. [N/A]
b.i. [N/A]
b.ii. [N/A]
c. [N/A]

Let $f(x) = \frac{1}{2}x^2 + kx + 8$, where $k \in \mathbb{Z}$.

a. Find the values of k such that $f(x) = 0$ has two equal roots. [4]

b. Each value of k is equally likely for $-5 \le k \le 5$. Find the probability that $f(x) = 0$ has no roots. [4]

Markscheme

a. **METHOD 1**

evidence of discriminant ***(M1)***

e.g. $b^2 - 4ac$, discriminant = 0

correct substitution into discriminant ***A1***

e.g. $k^2 - 4 \times \frac{1}{2} \times 8$, $k^2 - 16 = 0$

$k = \pm 4$ ***A1A1*** ***N3***

METHOD 2

recognizing that equal roots means perfect square ***(R1)***

e.g. attempt to complete the square, $\frac{1}{2}(x^2 + 2kx + 16)$

correct working

e.g. $\frac{1}{2}(x+k)^2$, $\frac{1}{2}k^2 = 8$ ***A1***

$k = \pm 4$ ***A1A1*** ***N3***

[4 marks]

b. evidence of appropriate approach ***(M1)***

e.g. $b^2 - 4ac < 0$

correct working for k ***A1***

e.g. $-4 < k < 4$, $k^2 < 16$, list all correct values of k

$p = \frac{7}{11}$ ***A2*** ***N3***

[4 marks]

Examiners report

a. A good number of candidates were successful in using the discriminant to find the correct values of k in part (a), however, there were many who tried to use the quadratic formula without recognizing the significance of the discriminant.

b. Part (b) was very poorly done by nearly all candidates. Common errors included finding the wrong values for k, and not realizing that there were 11 possible values for k.

Let $f(x) = 3x - 2$ and $g(x) = \frac{5}{3x}$, for $x \neq 0$.

Let $h(x) = \frac{5}{x+2}$, for $x \geqslant 0$. The graph of h has a horizontal asymptote at $y = 0$.

a. Find $f^{-1}(x)$. [2]

b. Show that $\left(g \circ f^{-1}\right)(x) = \frac{5}{x+2}$. [2]

c(i). Find the y-intercept of the graph of h. [2]

c(ii). Hence, sketch the graph of h. [3]

d(i). For the graph of h^{-1}, write down the x-intercept; [1]

d(ii). For the graph of h^{-1}, write down the equation of the vertical asymptote. [1]

e. Given that $h^{-1}(a) = 3$, find the value of a. [3]

Markscheme

a. interchanging x and y ***(M1)***

eg $x = 3y - 2$

$f^{-1}(x) = \frac{x+2}{3}$ $\left(\text{accept } y = \frac{x+2}{3},\ \frac{x+2}{3}\right)$ ***A1 N2***

[2 marks]

b. attempt to form composite (in any order) ***(M1)***

eg $g\left(\frac{x+2}{3}\right),\ \frac{\frac{5}{3x}+2}{3}$

correct substitution ***A1***

eg $\frac{5}{3\left(\frac{x+2}{3}\right)}$

$\left(g \circ f^{-1}\right)(x) = \frac{5}{x+2}$ ***AG N0***

[2 marks]

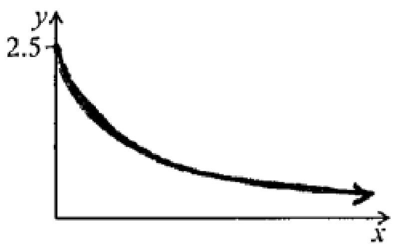
c(i). valid approach ***(M1)***

eg $h(0),\ \frac{5}{0+2}$

$y = \frac{5}{2}$ (accept $(0, 2.5)$) ***A1 N2***

[2 marks]

c(ii).



A1A2 N3

Notes: Award ***A1*** for approximately correct shape (reciprocal, decreasing, concave up).

Only if this ***A1*** is awarded, award ***A2*** for all the following approximately correct features: *y*-intercept at $(0, 2.5)$, asymptotic to *x*-axis, correct domain $x \geqslant 0$.

If only two of these features are correct, award ***A1***.

[3 marks]

d(i). $x = \frac{5}{2}$ (accept $(2.5, 0)$) ***A1 N1***

[1 mark]

d(ii). $x = 0$ (must be an equation) ***A1 N1***

[1 mark]

e. **METHOD 1**

attempt to substitute 3 into h (seen anywhere) ***(M1)***

eg $h(3),\ \frac{5}{3+2}$

correct equation ***(A1)***

eg $a = \frac{5}{3+2},\ h(3) = a$

$a = 1$ ***A1 N2***

[3 marks]

METHOD 2

attempt to find inverse (may be seen in (d)) ***(M1)***

eg $x = \frac{5}{y+2},\ h^{-1} = \frac{5}{x} - 2,\ \frac{5}{x} + 2$

correct equation, $\frac{5}{x} - 2 = 3$ ***(A1)***

$a = 1$ ***A1*** ***N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c(i). [N/A]
c(ii). [N/A]
d(i). [N/A]
d(ii). [N/A]
e. [N/A]

Let $f(x) = p + \frac{9}{x-q}$, for $x \neq q$. The line $x = 3$ is a vertical asymptote to the graph of f.

a. Write down the value of q. [1]

b. The graph of f has a y-intercept at $(0,\ 4)$. [4]

Find the value of p.

c. The graph of f has a y-intercept at $(0,\ 4)$. [1]

Write down the equation of the horizontal asymptote of the graph of f.

Markscheme

a. $q = 3$ ***A1*** ***N1***

[1 mark]

b. correct expression for $f(0)$ ***(A1)***

eg $p + \frac{9}{0-3}$, $4 = p + \frac{9}{-q}$

recognizing that $f(0) = 4$ (may be seen in equation) ***(M1)***

correct working ***(A1)***

eg $4 = p - 3$

$p = 7$ ***A1*** ***N3***

[3 marks]

c. $y = 7$ (must be an equation, do not accept $p = 7$ ***A1*** ***N1***

[1 mark]

Total [6 marks]

Examiners report

a. Parts (a) and (b) were generally well done. Some candidates incorrectly answered $q = -3$, rather than $q = 3$, in part (a), but then were able to earn follow-through marks in part (b).

b. Parts (a) and (b) were generally well done. Some candidates incorrectly answered $q = -3$, rather than $q = 3$, in part (a), but then were able to earn follow-through marks in part (b).

c. Many candidates did not recognize the connection between parts (b) and (c) of this question, and many did a good deal of unnecessary work in part (c) before giving the correct answer. In part (c), many candidates did not write the equation of the asymptote, but just wrote the number.

Let $f(x) = px^3 + px^2 + qx$.

a. Find $f'(x)$. [2]

b. Given that $f'(x) \geqslant 0$, show that $p^2 \leqslant 3pq$. [5]

Markscheme

a. $f'(x) = 3px^2 + 2px + q$ ***A2 N2***

Note: Award ***A1*** if only 1 error.

[2 marks]

b. evidence of discriminant (must be seen explicitly, not in quadratic formula) ***(M1)***

eg $b^2 - 4ac$

correct substitution into discriminant (may be seen in inequality) ***A1***

eg $(2p)^2 - 4 \times 3p \times q,\ 4p^2 - 12pq$

$f'(x) \geqslant 0$ then f' has two equal roots or no roots ***(R1)***

recognizing discriminant less or equal than zero ***R1***

eg $\Delta \leqslant 0,\ 4p^2 - 12pq \leqslant 0$

correct working that clearly leads to the required answer ***A1***

eg $p^2 - 3pq \leqslant 0,\ 4p^2 \leqslant 12pq$

$p^2 \leqslant 3pq$ ***AG N0***

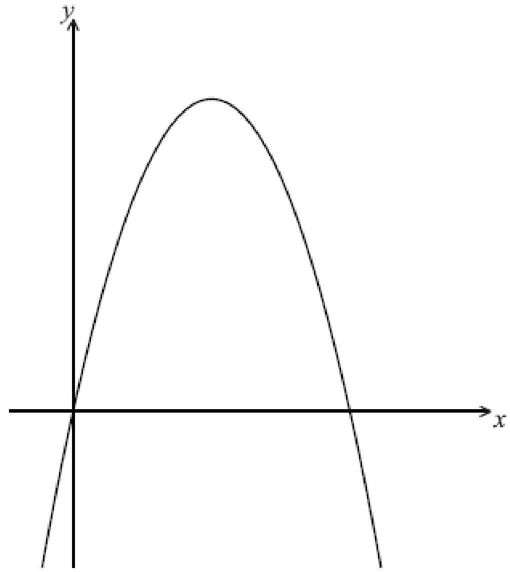
[5 marks]

Examiners report

a. [N/A]

b.

Let $f(x) = 8x - 2x^2$. Part of the graph of *f* is shown below.



a. Find the x-intercepts of the graph. [4]

b(i) and (ii). (i) Write down the equation of the axis of symmetry. [3]

(ii) Find the y-coordinate of the vertex.

Markscheme

a. evidence of setting function to zero ***(M1)***

e.g. $f(x) = 0$, $8x = 2x^2$

evidence of correct working ***A1***

e.g. $0 = 2x(4 - x)$, $\frac{-8 \pm \sqrt{64}}{-4}$

x-intercepts are at 4 and 0 (accept (4, 0) and (0, 0) , or $x = 4$, $x = 0$) ***A1A1*** ***N1N1***

[4 marks]

b(i) and (ii). (i) $x = 2$ (must be equation) ***A1*** ***N1***

(ii) substituting $x = 2$ into $f(x)$ ***(M1)***

$y = 8$ ***A1*** ***N2***

[3 marks]

Examiners report

a. This question was answered well by most candidates.

b(i) and (ii). This question was answered well by most candidates. Some did not give an equation for their axis of symmetry.

The velocity v ms^{-1} of a particle at time t seconds, is given by $v = 2t + \cos 2t$, for $0 \le t \le 2$.

a. Write down the velocity of the particle when $t = 0$. [1]

b(i) and (ii). When $t = k$, the acceleration is zero. [8]

(i) Show that $k = \frac{\pi}{4}$.

(ii) Find the exact velocity when $t = \frac{\pi}{4}$.

c. When $t < \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$ and when $t > \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$. [4]

Sketch a graph of *v* against *t* .

d(i) and (ii). Let *d* be the distance travelled by the particle for $0 \le t \le 1$. [3]

(i) Write down an expression for *d* .

(ii) Represent *d* on your sketch.

Markscheme

a. $v = 1$ ***A1*** ***N1***

[1 mark]

b(i) and (ii). (i) $\frac{\mathrm{d}}{\mathrm{d}t}(2t) = 2$ ***A1***

$\frac{\mathrm{d}}{\mathrm{d}t}(\cos 2t) = -2\sin 2t$ ***A1A1***

Note: Award ***A1*** for coefficient 2 and ***A1*** for $-\sin 2t$.

evidence of considering acceleration = 0 ***(M1)***

e.g. $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$, $2 - 2\sin 2t = 0$

correct manipulation ***A1***

e.g. $\sin 2k = 1$, $\sin 2t = 1$

$2k = \frac{\pi}{2}$ (accept $2t = \frac{\pi}{2}$) ***A1***

$k = \frac{\pi}{4}$ ***AG*** ***N0***

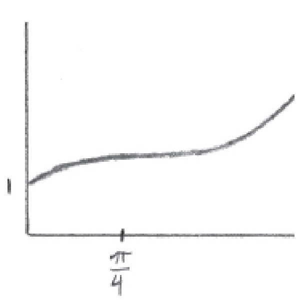
(ii) attempt to substitute $t = \frac{\pi}{4}$ into *v* ***(M1)***

e.g. $2\left(\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{4}\right)$

$v = \frac{\pi}{2}$ ***A1*** ***N2***

[8 marks]

c.



A1A1A2 ***N4***

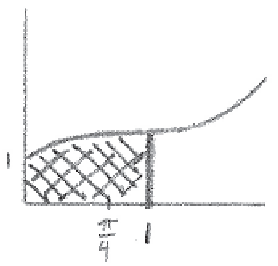
Notes: Award ***A1*** for *y*-intercept at $(0, 1)$, ***A1*** for curve having zero gradient at $t = \frac{\pi}{4}$, ***A2*** for shape that is concave down to the left of $\frac{\pi}{4}$ and concave up to the right of $\frac{\pi}{4}$. If a correct curve is drawn without indicating $t = \frac{\pi}{4}$, do not award the second ***A1*** for the zero gradient, but award the final ***A2*** if appropriate. Sketch need not be drawn to scale. Only essential features need to be clear.

[4 marks]

d(i) and (ii). (a) correct expression ***A2***

e.g. $\int_0^1 (2t + \cos 2t)\mathrm{d}t$, $\left[t^2 + \frac{\sin 2t}{2}\right]_0^1$, $1 + \frac{\sin 2}{2}$, $\int_0^1 v\mathrm{d}t$

(ii)



A1

Note: The line at $t = 1$ needs to be clearly after $t = \frac{\pi}{4}$.

[3 marks]

Examiners report

a. Many candidates gave a correct initial velocity, although a substantial number of candidates answered that $0 + \cos 0 = 0$.

b(i) and (ii). For (i), students commonly applied the chain rule correctly to achieve the derivative, and many recognized that the acceleration must be zero. Occasionally a student would use a double-angle identity on the velocity function before differentiating. This is not incorrect, but it usually caused problems when trying to show $k = \frac{\pi}{4}$. At times students would reach the equation $\sin 2k = 1$ and then substitute the $\frac{\pi}{4}$, which does not satisfy the “show that” instruction.

c. The challenge in this question is sketching the graph using the information achieved and provided. This requires students to make graphical interpretations, and as typical in section B, to link the early parts of the question with later parts. Part (a) provides the *y*-intercept, and part (b) gives a point with a horizontal tangent. Plotting these points first was a helpful strategy. Few understood either the notation or the concept that the function had to be increasing on either side of the $\frac{\pi}{4}$, with most thinking that the point was either a max or min. It was the astute student who recognized that the derivatives being positive on either side of $\frac{\pi}{4}$ creates a point of inflexion.

Additionally, important points should be labelled in a sketch. Indicating the $\frac{\pi}{4}$ on the *x*-axis is a requirement of a clear graph. Although students were not penalized for not labelling the $\frac{\pi}{2}$ on the *y*-axis, there should be a recognition that the point is higher than the *y*-intercept.

d(i) and (ii). While some candidates recognized that the distance is the area under the velocity graph, surprisingly few included neither the limits of integration in their expression, nor the “d*t*”. Most unnecessarily attempted to integrate the function, often giving an answer with “+C”, and only earned marks if the limits were included with their result. Few recognized that a shaded area is an adequate representation of distance on the sketch, with most fruitlessly attempting to graph a new curve.

Let $f(x) = px^2 + (10 - p)x + \frac{5}{4}p - 5$.

a. Show that the discriminant of $f(x)$ is $100 - 4p^2$. [3]

b. Find the values of p so that $f(x) = 0$ has two **equal** roots. [3]

Markscheme

a. correct substitution into $b^2 - 4ac$ ***A1***

eg $(10-p)^2 - 4(p)\left(\frac{5}{4}p - 5\right)$

correct expansion of each term ***A1A1***

eg $100 - 20p + p^2 - 5p^2 + 20p,\ 100 - 20p + p^2 - (5p^2 - 20p)$

$100 - 4p^2$ ***AG N0***

[3 marks]

b. recognizing discriminant is zero for equal roots ***(R1)***

eg $D = 0,\ 4p^2 = 100$

correct working ***(A1)***

eg $p^2 = 25$, 1 correct value of p

both correct values $p = \pm 5$ ***A1 N2***

[3 marks]

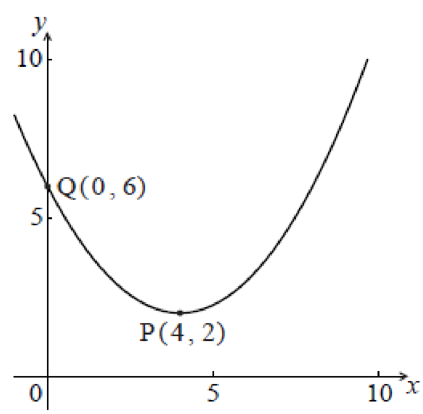
Total [6 marks]

Examiners report

a. Many candidates were able to identify the discriminant correctly and continued with good algebraic manipulation. A commonly seen mistake was identifying the constant as $\frac{5}{4}p$ instead of $\frac{5}{4}p - 5$. Mostly a correct approach to part b) was seen $(\Delta = 0)$, with the common error being only one answer given for p, even though the question said values (plural).

b. Many candidates were able to identify the discriminant correctly and continued with good algebraic manipulation. A commonly seen mistake was identifying the constant as $\frac{5}{4}p$ instead of $\frac{5}{4}p - 5$. Mostly a correct approach to part b) was seen $(\Delta = 0)$, with the common error being only one answer given for p, even though the question said values (plural).

Let f be a quadratic function. Part of the graph of f is shown below.



The vertex is at P(4, 2) and the *y*-intercept is at Q(0, 6) .

a. Write down the equation of the axis of symmetry. [1]

b. The function *f* can be written in the form $f(x) = a(x - h)^2 + k$. [2]

Write down the value of *h* and of *k* .

c. The function *f* can be written in the form $f(x) = a(x - h)^2 + k$. [3]

Find *a* .

Markscheme

a. $x = 4$ (must be an equation) ***A1 N1***

[1 mark]

b. $h = 4$, $k = 2$ ***A1A1 N2***

[2 marks]

c. attempt to substitute coordinates of any point on the graph into *f* ***(M1)***

e.g. $f(0) = 6$, $6 = a(0 - 4)^2 + 2$, $f(4) = 2$

correct equation (do **not** accept an equation that results from $f(4) = 2$) ***(A1)***

e.g. $6 = a(-4)^2 + 2$, $6 = 16a + 2$

$a = \frac{4}{16}\left(= \frac{1}{4}\right)$ ***A1 N2***

[3 marks]

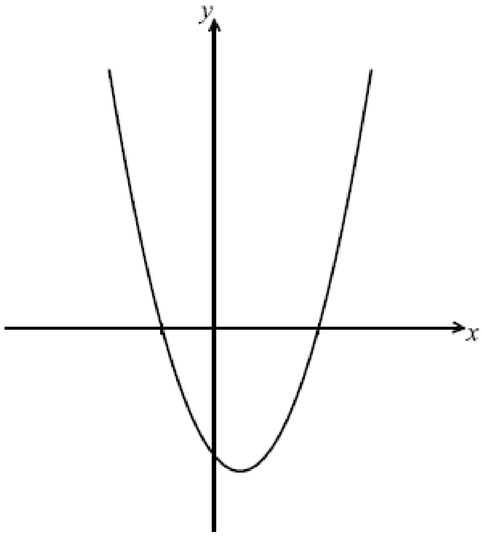
Examiners report

a. A surprising number of candidates missed part (a) of this question, which required them to write the equation of the axis of symmetry. Some candidates did not write their answer as an equation, while others simply wrote the formula $x = -\frac{b}{2a}$.

b. This was answered correctly by the large majority of candidates.

c. The rest of this question was answered correctly by the large majority of candidates. The mistakes seen in part (c) were generally due to either incorrect substitution of a point into the equation, or substitution of the vertex coordinates, which got the candidates nowhere.

The following diagram shows part of the graph of *f* , where $f(x) = x^2 - x - 2$.



a. Find both x-intercepts. [4]

b. Find the x-coordinate of the vertex. [2]

Markscheme

a. evidence of attempting to solve $f(x) = 0$ ***(M1)***

evidence of correct working ***A1***

e.g. $(x+1)(x-2)$, $\frac{1 \pm \sqrt{9}}{2}$

intercepts are $(-1, 0)$ and $(2, 0)$ (accept $x = -1$, $x = 2$) ***A1A1 N1N1***

[4 marks]

b. evidence of appropriate method ***(M1)***

e.g. $x_v = \frac{x_1 + x_2}{2}$, $x_v = -\frac{b}{2a}$, reference to symmetry

$x_v = 0.5$ ***A1 N2***

[2 marks]

Examiners report

a. This question was consistently the best handled one on the entire paper.

b. This question was consistently the best handled one on the entire paper.

Consider $f(x) = \ln(x^4 + 1)$.

The second derivative is given by $f''(x) = \frac{4x^2(3 - x^4)}{(x^4+1)^2}$.

The equation $f''(x) = 0$ has only three solutions, when $x = 0$, $\pm\sqrt[4]{3}$ $(\pm 1.316\ldots)$.

a. Find the value of $f(0)$. [2]

b. Find the set of values of x for which f is increasing. [5]

c. (i) Find $f''(1)$. [5]

(ii) **Hence**, show that there is no point of inflexion on the graph of f at $x = 0$.

d. There is a point of inflexion on the graph of f at $x = \sqrt[4]{3}$ $(x = 1.316\ldots)$. [3]

Sketch the graph of f , for $x \geq 0$.

Markscheme

a. substitute 0 into f ***(M1)***

eg $\ln(0+1)$, $\ln 1$

$f(0) = 0$ ***A1 N2***

[2 marks]

b. $f'(x) = \frac{1}{x^4+1} \times 4x^3$ (seen anywhere) ***A1A1***

Note: Award ***A1*** for $\frac{1}{x^4+1}$ and ***A1*** for $4x^3$.

recognizing f increasing where $f'(x) > 0$ (seen anywhere) ***R1***

eg $f'(x) > 0$, diagram of signs

attempt to solve $f'(x) > 0$ ***(M1)***

eg $4x^3 = 0$, $x^3 > 0$

f increasing for $x > 0$ (accept $x \geq 0$) ***A1 N1***

[5 marks]

c. (i) substituting $x = 1$ into f'' ***(A1)***

eg $\frac{4(3-1)}{(1+1)^2}$, $\frac{4\times2}{4}$

$f''(1) = 2$ ***A1 N2***

(ii) valid interpretation of point of inflexion (seen anywhere) ***R1***

eg no change of sign in $f''(x)$, no change in concavity,

f' increasing both sides of zero

attempt to find $f''(x)$ for $x < 0$ ***(M1)***

eg $f''(-1)$, $\frac{4(-1)^2(3-(-1)^4)}{((-1)^4+1)^2}$, diagram of signs

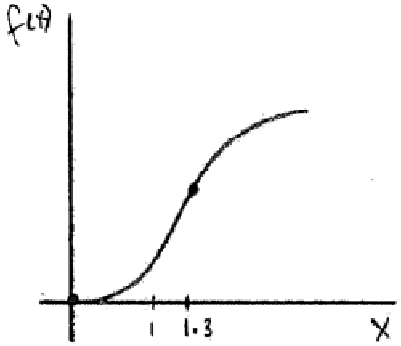
correct working leading to positive value ***A1***

eg $f''(-1) = 2$, discussing signs of numerator **and** denominator

there is no point of inflexion at $x = 0$ ***AG N0***

[5 marks]

d.



A1A1A1 *N3*

Notes: Award *A1* for shape concave up left of POI and concave down right of POI.

Only if this *A1* is awarded, then award the following:

A1 for curve through (0, 0) , *A1* for increasing throughout.

Sketch need not be drawn to scale. Only essential features need to be clear.

[3 marks]

Examiners report

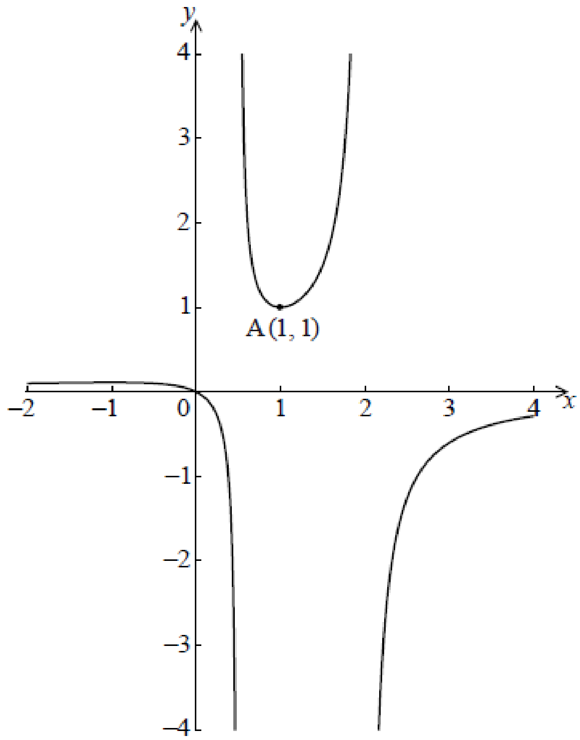
a. Many candidates left their answer to part (a) as $\ln 1$. While this shows an understanding for substituting a value into a function, it leaves an unfinished answer that should be expressed as an integer.

b. Candidates who attempted to consider where f is increasing generally understood the derivative is needed. However, a number of candidates did not apply the chain rule, which commonly led to answers such as "increasing for all x". Many set their derivative equal to zero, while neglecting to indicate in their working that $f'(x) > 0$ for an increasing function. Some created a diagram of signs, which provides appropriate evidence as long as it is clear that the signs represent f'.

c. Finding $f''(1)$ proved no challenge, however, using this value to **show that** no point of inflexion exists proved elusive for many. Some candidates recognized the signs must not change in the second derivative. Few candidates presented evidence in the form of a calculation, which follows from the "hence" command of the question. In this case, a sign diagram without numerical evidence was not sufficient.

d. Few candidates created a correct graph from the information given or found in the question. This included the point (0, 0), the fact that the function is always increasing for $x > 0$, the concavity at $x = 1$ and the change in concavity at the given point of inflexion. Many incorrect attempts showed a graph concave down to the right of $x = 0$, changing to concave up.

Let $f(x) = \frac{x}{-2x^2+5x-2}$ for $-2 \le x \le 4$, $x \ne \frac{1}{2}$, $x \ne 2$. The graph of f is given below.



The graph of f has a local minimum at A(1, 1) and a local maximum at B.

a. Use the quotient rule to show that $f'(x) = \frac{2x^2-2}{(-2x^2+5x-2)^2}$. [6]

b. Hence find the coordinates of B. [7]

c. Given that the line $y = k$ does not meet the graph of f, find the possible values of k . [3]

Markscheme

a. correct derivatives **applied** in quotient rule ***(A1)A1A1***

$1, -4x + 5$

Note: Award ***(A1)*** for 1, ***A1*** for $-4x$ and ***A1*** for 5, **only** if it is clear candidates are using the quotient rule.

correct substitution into quotient rule ***A1***

e.g. $\frac{1\times(-2x^2+5x-2)-x(-4x+5)}{(-2x^2+5x-2)^2}$, $\frac{-2x^2+5x-2-x(-4x+5)}{(-2x^2+5x-2)^2}$

correct working ***(A1)***

e.g. $\frac{-2x^2+5x-2-(-4x^2+5x)}{(-2x^2+5x-2)^2}$

expression clearly leading to the answer ***A1***

e.g. $\frac{-2x^2+5x-2+4x^2-5x}{(-2x^2+5x-2)^2}$

$f'(x) = \frac{2x^2-2}{(-2x^2+5x-2)^2}$ ***AG N0***

[6 marks]

b. evidence of attempting to solve $f'(x) = 0$ ***(M1)***

e.g. $2x^2 - 2 = 0$

evidence of correct working ***A1***

e.g. $x^2 = 1, \frac{\pm\sqrt{16}}{4}, 2(x-1)(x+1)$

correct solution to quadratic ***(A1)***

e.g. $x = \pm 1$

correct *x*-coordinate $x = -1$ (may be seen in coordinate form $\left(-1, \frac{1}{9}\right)$) ***A1 N2***

attempt to substitute -1 into *f* (do not accept any other value) ***(M1)***

e.g. $f(-1) = \frac{-1}{-2\times(-1)^2+5\times(-1)-2}$

correct working

e.g. $\frac{-1}{-2-5-2}$ ***A1***

correct *y*-coordinate $y = \frac{1}{9}$ (may be seen in coordinate form $\left(-1, \frac{1}{9}\right)$) ***A1 N2***

[7 marks]

c. recognizing values between max and min ***(R1)***

$\frac{1}{9} < k < 1$ ***A2 N3***

[3 marks]

Examiners report

a. While most candidates answered part (a) correctly, there were some who did not show quite enough work for a "show that" question. A very small number of candidates did not follow the instruction to use the quotient rule.

b. In part (b), most candidates knew that they needed to solve the equation $f'(x) = 0$, and many were successful in answering this question correctly. However, some candidates failed to find both values of *x*, or made other algebraic errors in their solutions. One common error was to find only one solution for $x^2 = 1$; another was to work with the denominator equal to zero, rather than the numerator.

c. In part (c), a significant number of candidates seemed to think that the line $y = k$ was a vertical line, and attempted to find the vertical asymptotes. Others tried looking for a horizontal asymptote. Fortunately, there were still a good number of intuitive candidates who recognized the link with the graph and with part (b), and realized that the horizontal line must pass through the space between the given local minimum and the local maximum they had found in part (b).

---

a. Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n. [2]

b. Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$. [4]

Markscheme

a. $m = 3,\ n = 4$ ***A1A1 N2***

[2 marks]

b. attempt to apply $(2^a)^b = 2^{ab}$ ***(M1)*** *eg* $6x+3,\ 4(2x-3)$

equating **their** powers of 2 (seen anywhere) ***M1***

eg $3(2x+1)=8x-12$

correct working ***A1***

eg $8x-12=6x+3,\ 2x=15$

$x=\frac{15}{2}\quad(7.5)$ ***A1*** ***N2***

[4 marks]

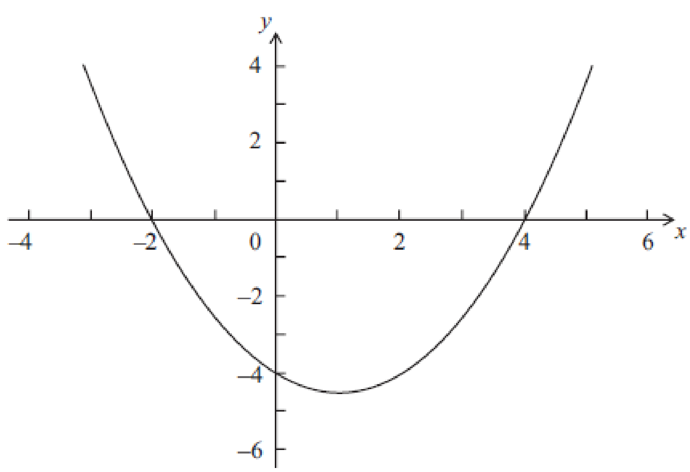
Total [6 marks]

Examiners report

a. Indices laws were well understood with many candidates solving the equation correctly. Some candidates used logs, which took longer, and errors crept in.

b. Indices laws were well understood with many candidates solving the equation correctly. Some candidates used logs, which took longer, and errors crept in.

Let $f(x)=p(x-q)(x-r)$. Part of the graph of f is shown below.



The graph passes through the points $(-2,\ 0)$, $(0,\ -4)$ and $(4,\ 0)$.

a. Write down the value of q and of r. [2]

b. Write down the **equation** of the axis of symmetry. [1]

c. Find the value of p. [3]

Markscheme

a. $q=-2$, $r=4$ or $q=4$, $r=-2$ ***A1A1*** ***N2***

[2 marks]

b. $x = 1$ (must be an equation) ***A1*** ***N1***

[1 mark]

c. substituting $(0,\ -4)$ into the equation ***(M1)***

e.g. $-4 = p(0-(-2))(0-4)$, $-4 = p(-4)(2)$

correct working towards solution ***(A1)***

e.g. $-4 = -8p$

$p = \frac{4}{8}\left(=\frac{1}{2}\right)$ ***A1*** ***N2***

[3 marks]

Examiners report

a. The majority of candidates were successful on some or all parts of this question, with some candidates using a mix of algebra and graphical reasoning and others ignoring the graph and working only algebraically. Some did not recognize that p and q are the roots of the quadratic function and hence gave the answers as 2 and -4.

b. A common error in part (b) was the absence of an equation. Some candidates wrote down the equation $x = \frac{-b}{2a}$ but were not able to substitute correctly. Those students did not realize that the axis of symmetry is always halfway between the x-intercepts.

c. More candidates had trouble with part (c) with erroneous substitutions and simplification mistakes commonplace.

Let $f(x) = 6x\sqrt{1-x^2}$, for $-1 \leqslant x \leqslant 1$, and $g(x) = \cos(x)$, for $0 \leqslant x \leqslant \pi$.

Let $h(x) = (f \circ g)(x)$.

a. Write $h(x)$ in the form $a\sin(bx)$, where $a,\ b \in \mathbb{Z}$. [5]

b. Hence find the range of h. [2]

Markscheme

a. attempt to form composite in any order ***(M1)***

eg $f\left(g(x)\right),\ \cos\left(6x\sqrt{1-x^2}\right)$

correct working ***(A1)***

eg $6\cos x\sqrt{1-\cos^2 x}$

correct application of Pythagorean identity (do not accept $\sin^2 x + \cos^2 x = 1$) ***(A1)***

eg $\sin^2 x = 1-\cos^2 x,\ 6\cos x\sin x,\ 6\cos x\left|\sin x\right|$

valid approach (do not accept $2\sin x\cos x = \sin 2x$) ***(M1)***

eg $3(2\cos x\sin x)$

$h(x) = 3\sin 2x$ ***A1*** ***N3***

[5 marks]

b. valid approach ***(M1)***

eg amplitude $= 3$, sketch with max and min y-values labelled, $-3 < y < 3$

correct range ***A1*** ***N2***

eg $-3 \leqslant y \leqslant 3$, $[-3,\ 3]$ from -3 to 3

Note: Do not award ***A1*** for $-3 < y < 3$ or for "between -3 and 3".

[2 marks]

Examiners report

a. In part (a), nearly all candidates found the correct composite function in terms of $\cos x$, though many did not get any further than this first step in their solution to the question. While some candidates seemed to recognize the need to use trigonometric identities, most were unsuccessful in finding the correct expression in the required form.

b. In part (b), very few candidates were able to provide the correct range of the function.

A quadratic function f can be written in the form $f(x) = a(x - p)(x - 3)$. The graph of f has axis of symmetry $x = 2.5$ and y-intercept at $(0,\ -6)$

a. Find the value of p. [3]

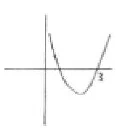
b. Find the value of a. [3]

c. The line $y = kx - 5$ is a tangent to the curve of f. Find the values of k. [8]

Markscheme

a. **METHOD 1 (using *x*-intercept)**

determining that 3 is an x-intercept ***(M1)***

eg $x - 3 = 0$, 

valid approach ***(M1)***

eg $3 - 2.5$, $\frac{p+3}{2} = 2.5$

$p = 2$ ***A1*** ***N2***

METHOD 2 (expanding *f* (*x*))

correct expansion (accept absence of a) ***(A1)***

eg $ax^2 - a(3 + p)x + 3ap,\ x^2 - (3 + p)x + 3p$

valid approach involving equation of axis of symmetry ***(M1)***

eg $\frac{-b}{2a} = 2.5,\ \frac{a(3+p)}{2a} = \frac{5}{2},\ \frac{3+p}{2} = \frac{5}{2}$

$p = 2$ ***A1*** ***N2***

METHOD 3 (using derivative)

correct derivative (accept absence of a) ***(A1)***

eg $a(2x-3-p),\ 2x-3-p$

valid approach ***(M1)***

eg $f'(2.5)=0$

$p=2$ ***A1 N2***

[3 marks]

b. attempt to substitute $(0,\ -6)$ ***(M1)***

eg $-6=a(0-2)(0-3),\ 0=a(-8)(-9),\ a(0)^2-5a(0)+6a=-6$

correct working ***(A1)***

eg $-6=6a$

$a=-1$ ***A1 N2***

[3 marks]

c. **METHOD 1 (using discriminant)**

recognizing tangent intersects curve once ***(M1)***

recognizing one solution when discriminant = 0 ***M1***

attempt to set up equation ***(M1)***

eg $g=f,\ kx-5=-x^2+5x-6$

rearranging their equation to equal zero ***(M1)***

eg $x^2-5x+kx+1=0$

correct discriminant (if seen explicitly, not just in quadratic formula) ***A1***

eg $(k-5)^2-4,\ 25-10k+k^2-4$

correct working ***(A1)***

eg $k-5=\pm2,\ (k-3)(k-7)=0,\ \frac{10\pm\sqrt{100-4\times21}}{2}$

$k=3,\ 7$ ***A1A1 N0***

METHOD 2 (using derivatives)

attempt to set up equation ***(M1)***

eg $g=f,\ kx-5=-x^2+5x-6$

recognizing derivative/slope are equal ***(M1)***

eg $f'=m_T,\ f'=k$

correct derivative of f ***(A1)***

eg $-2x+5$

attempt to set up equation in terms of either x or k ***M1***

eg $(-2x+5)x-5=-x^2+5x-6,\ k\left(\frac{5-k}{2}\right)-5=-\left(\frac{5-k}{2}\right)^2+5\left(\frac{5-k}{2}\right)-6$

rearranging their equation to equal zero ***(M1)***

eg $x^2-1=0,\ k^2-10k+21=0$

correct working ***(A1)***

eg $x=\pm1,\ (k-3)(k-7)=0,\ \frac{10\pm\sqrt{100-4\times21}}{2}$

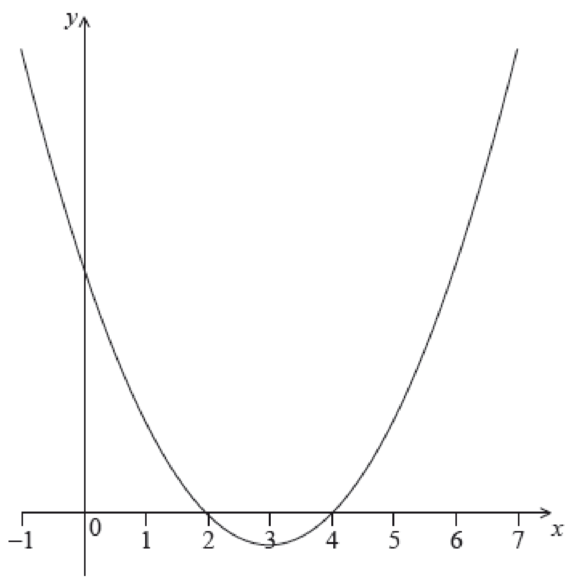
$k=3,\ 7$ ***A1A1 N0***

[8 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The following diagram shows part of the graph of a quadratic function f.



The vertex is at $(3,\ -1)$ and the x-intercepts at 2 and 4.

The function f can be written in the form $f(x) = (x-h)^2 + k$.

The function can also be written in the form $f(x) = (x-a)(x-b)$.

a. Write down the value of h and of k. [2]

b. Write down the value of a and of b. [2]

c. Find the y-intercept. [2]

Markscheme

a. $h = 3,\ k = -1$ ***A1A1*** ***N2***

[2 marks]

b. $a = 2,\ b = 4\ (\text{or } a = 4,\ b = 2)$ ***A1A1*** ***N2***

[2 marks]

c. attempt to substitute $x = 0$ into their f ***(M1)***

eg $(0-3)^2 - 1,\ (0-2)(0-4)$

$y = 8$ ***A1*** ***N2***

[2 marks]

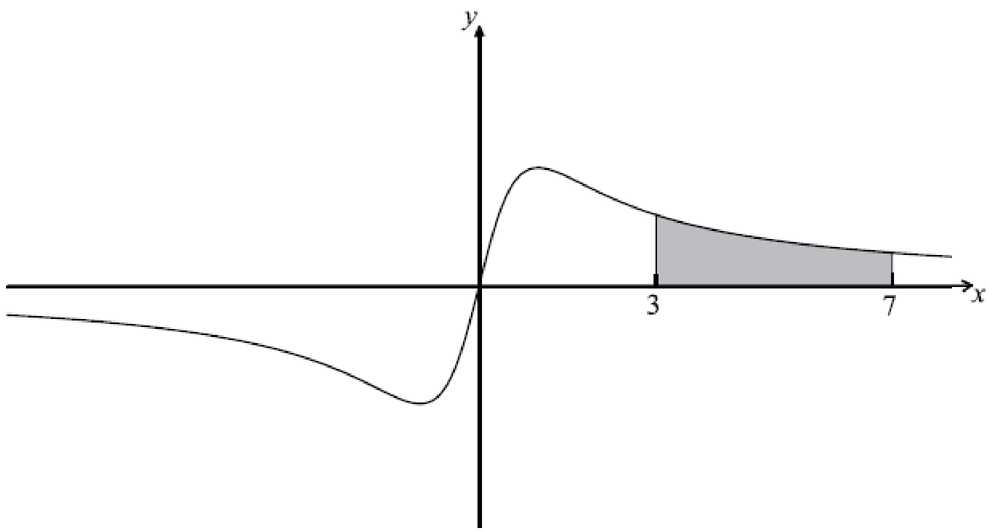
Examiners report

a. Nearly all candidates performed well on this question, earning full marks on all three question parts.

b. Nearly all candidates performed well on this question, earning full marks on all three question parts. In part (b), there were some candidates who factored the quadratic expression correctly, but went on to give negative values for a and b.

c. Nearly all candidates performed well on this question, earning full marks on all three question parts.

Let $f(x) = \frac{ax}{x^2+1}$, $-8 \le x \le 8$, $a \in \mathbb{R}$ .The graph of f is shown below.



The region between $x = 3$ and $x = 7$ is shaded.

a. Show that $f(-x) = -f(x)$. [2]

b. Given that $f''(x) = \frac{2ax(x^2-3)}{(x^2+1)^3}$, find the coordinates of all points of inflexion. [7]

c. It is given that $\int f(x)\mathrm{d}x = \frac{a}{2}\ln(x^2+1) + C$. [7]

(i) Find the area of the shaded region, giving your answer in the form $p\ln q$.

(ii) Find the value of $\int_4^8 2f(x-1)\mathrm{d}x$.

Markscheme

a. **METHOD 1**

evidence of substituting $-x$ for x ***(M1)***

$f(-x) = \frac{a(-x)}{(-x)^2+1}$ ***A1***

$f(-x) = \frac{-ax}{x^2+1}$ $(= -f(x))$ ***AG*** ***N0***

METHOD 2

$y = -f(x)$ is reflection of $y = f(x)$ in x axis

and $y = f(-x)$ is reflection of $y = f(x)$ in y axis ***(M1)*** sketch showing these are the same ***A1***

$f(-x)=\frac{-ax}{x^2+1}$ $(=-f(x))$ ***AG*** ***N0***

[2 marks]

b. evidence of appropriate approach ***(M1)***

e.g. $f''(x)=0$

to set the numerator equal to 0 ***(A1)***

e.g. $2ax(x^2-3)=0$; $(x^2-3)=0$

$(0, 0)$, $\left(\sqrt{3}, \frac{a\sqrt{3}}{4}\right)$, $\left(-\sqrt{3}, -\frac{a\sqrt{3}}{4}\right)$ (accept $x=0$, $y=0$ etc) ***A1A1A1A1A1*** ***N5***

[7 marks]

c. (i) correct expression ***A2***

e.g. $\left[\frac{a}{2}\ln(x^2+1)\right]_3^7$, $\frac{a}{2}\ln 50-\frac{a}{2}\ln 10$, $\frac{a}{2}(\ln 50-\ln 10)$

area = $\frac{a}{2}\ln 5$ ***A1A1*** ***N2***

(ii) **METHOD 1**

recognizing the shift that does not change the area ***(M1)***

e.g. $\int_4^8 f(x-1)\mathrm{d}x=\int_3^7 f(x)\mathrm{d}x$, $\frac{a}{2}\ln 5$

recognizing that the factor of 2 doubles the area ***(M1)***

e.g. $\int_4^8 2f(x-1)\mathrm{d}x$ $=2\int_4^8 f(x-1)\mathrm{d}x$ $\left(=2\int_3^7 f(x)\mathrm{d}x\right)$

$\int_4^8 2f(x-1)\mathrm{d}x=a\ln 5$ (i.e. $2\times$ **their** answer to (c)(i)) ***A1*** ***N3***

METHOD 2

changing variable

let $w=x-1$, so $\frac{\mathrm{d}w}{\mathrm{d}x}=1$

$2\int f(w)\mathrm{d}w$ $=\frac{2a}{2}\ln(w^2+1)+c$ ***(M1)***

substituting correct limits

e.g. $\left[a\ln\left[(x-1)^2+1\right]\right]_4^8$, $\left[a\ln(w^2+1)\right]_3^7$, $a\ln 50-a\ln 10$ ***(M1)***

$\int_4^8 2f(x-1)\mathrm{d}x=a\ln 5$ ***A1*** ***N3***

[7 marks]

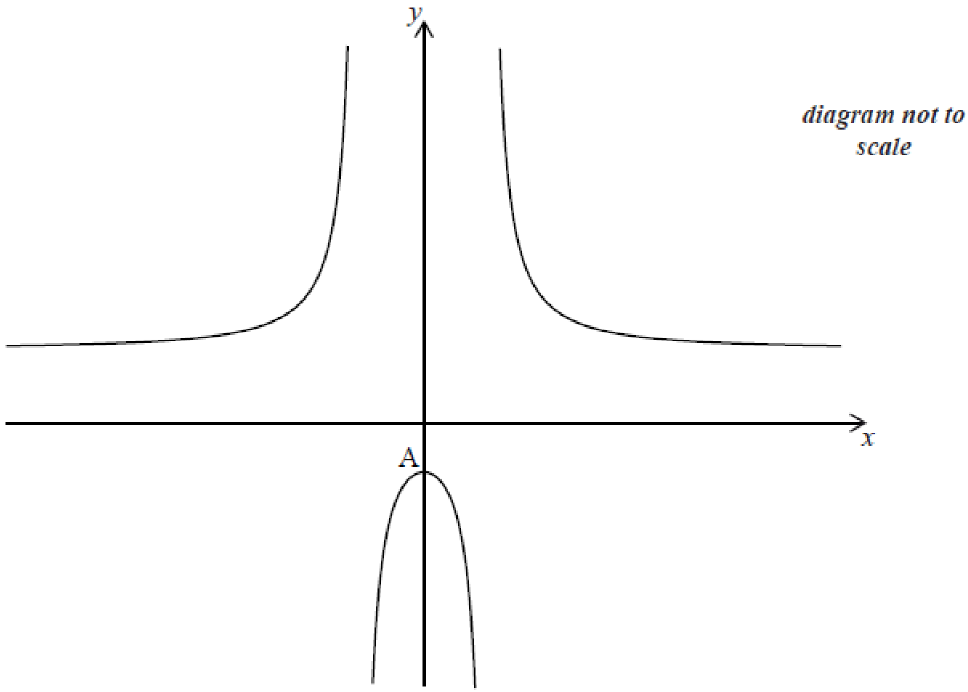
Examiners report

a. Part (a) was achieved by some candidates, although brackets around the $-x$ were commonly neglected. Some attempted to show the relationship by substituting a specific value for x . This earned no marks as a general argument is required.

b. Although many recognized the requirement to set the second derivative to zero in (b), a majority neglected to give their answers as ordered pairs, only writing the x-coordinates. Some did not consider the negative root.

c. For those who found a correct expression in (c)(i), many finished by calculating $\ln 50-\ln 10=\ln 40$. Few recognized that the translation did not change the area, although some factored the 2 from the integrand, appreciating that the area is double that in (c)(i).

Let $f(x) = 3 + \frac{20}{x^2-4}$, for $x \neq \pm 2$. The graph of *f* is given below.



The *y*-intercept is at the point A.

a. (i) Find the coordinates of A. [7]

(ii) Show that $f'(x) = 0$ at A.

b. The second derivative $f''(x) = \frac{40(3x^2+4)}{(x^2-4)^3}$. Use this to [6]

(i) justify that the graph of *f* has a local maximum at A;

(ii) explain why the graph of *f* does **not** have a point of inflexion.

c. Describe the behaviour of the graph of f for large $|x|$. [1]

d. Write down the range of f . [2]

Markscheme

a. (i) coordinates of A are $(0,\ -2)$ ***A1A1 N2***

(ii) derivative of $x^2 - 4 = 2x$ (seen anywhere) ***(A1)***

evidence of correct approach ***(M1)***

e.g. quotient rule, chain rule

finding $f'(x)$ ***A2***

e.g. $f'(x) = 20 \times (-1) \times (x^2-4)^{-2} \times (2x)$, $\frac{(x^2-4)(0)-(20)(2x)}{(x^2-4)^2}$

substituting $x = 0$ into $f'(x)$ (do not accept solving $f'(x) = 0$) ***M1***

at A $f'(x) = 0$ ***AG N0***

[7 marks]

b. (i) reference to $f'(x) = 0$ (seen anywhere) ***(R1)*** reference to $f''(0)$ is negative (seen anywhere) ***R1***

evidence of substituting $x = 0$ into $f''(x)$ ***M1***

finding $f''(0) = \frac{40\times4}{(-4)^3}\left(= -\frac{5}{2}\right)$ ***A1***

then the graph must have a local maximum ***AG***

(ii) reference to $f''(x) = 0$ at point of inflexion ***(R1)***

recognizing that the second derivative is never 0 ***A1*** ***N2***

e.g. $40(3x^2+4) \neq 0$, $3x^2+4 \neq 0$, $x^2 \neq -\frac{4}{3}$, the numerator is always positive

Note: Do not accept the use of the first derivative in part (b).

[6 marks]

c. correct (informal) statement, including reference to approaching $y = 3$ ***A1*** ***N1***

e.g. getting closer to the line $y = 3$, horizontal asymptote at $y = 3$

[1 mark]

d. **correct** inequalities, $y \leq -2$, $y > 3$, ***FT*** from (a)(i) and (c) ***A1A1*** ***N2***

[2 marks]

Examiners report

a. Almost all candidates earned the first two marks in part (a) (i), although fewer were able to apply the quotient rule correctly.

b. Many candidates were able to state how the second derivative can be used to identify maximum and inflection points, but fewer were actually able to demonstrate this with the given function. For example, in (b)(ii) candidates often simply said "the second derivative cannot equal 0" but did not justify or explain why this was true.

c. Not too many candidates could do part (c) correctly.

d. In (d) even those who knew what the range was had difficulty expressing the inequalities correctly.

Solve $\log_2 x + \log_2(x-2) = 3$, for $x > 2$.

Markscheme

recognizing $\log a + \log b = \log ab$ (seen anywhere) ***(A1)***

e.g. $\log_2(x(x-2))$, $x^2 - 2x$

recognizing $\log_a b = x \Leftrightarrow a^x = b$ ***(A1)***

e.g. $2^3 = 8$

correct simplification ***A1***

e.g. $x(x-2) = 2^3$, $x^2 - 2x - 8$

evidence of correct approach to solve ***(M1)***

e.g. factorizing, quadratic formula

correct working *A1*

e.g. $(x-4)(x+2)$, $\frac{2\pm\sqrt{36}}{2}$

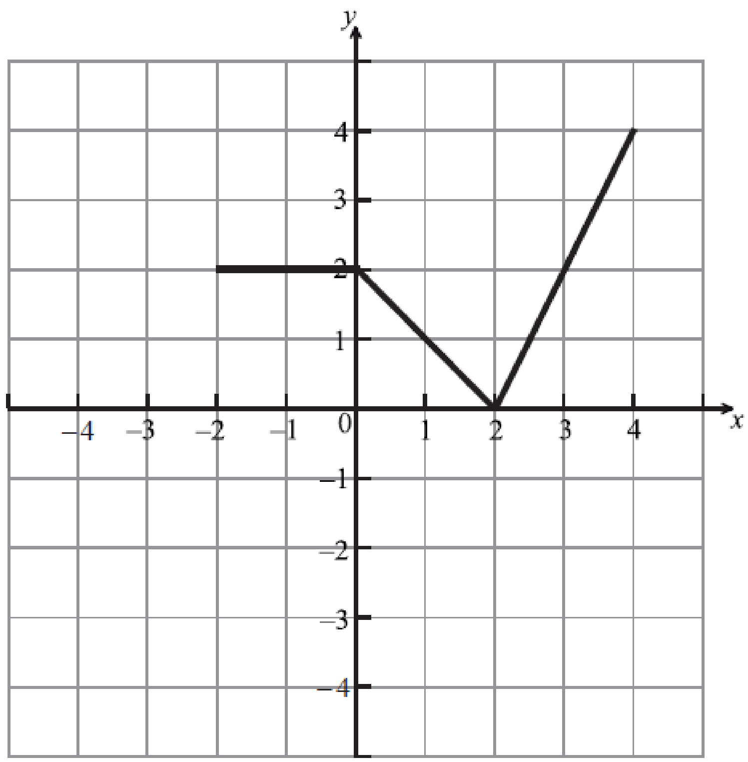
$x=4$ *A2* *N3*

[7 marks]

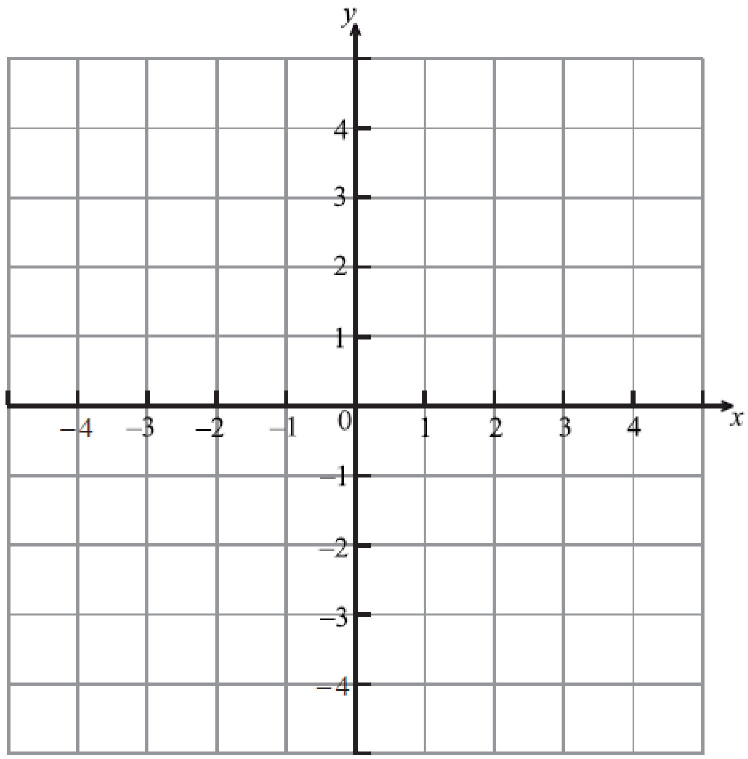
Examiners report

Candidates secure in their understanding of logarithm properties usually had success with this problem, solving the resulting quadratic either by factoring or using the quadratic formula. The majority of successful candidates correctly rejected the solution that was not in the domain. A number of candidates, however, were unclear on logarithm properties. Some unsuccessful candidates were able to demonstrate understanding of one property but without both were not able to make much progress. A few candidates employed a “guess and check” strategy, but this did not earn full marks.

The diagram below shows the graph of a function $f(x)$, for $-2\le x\le 4$.



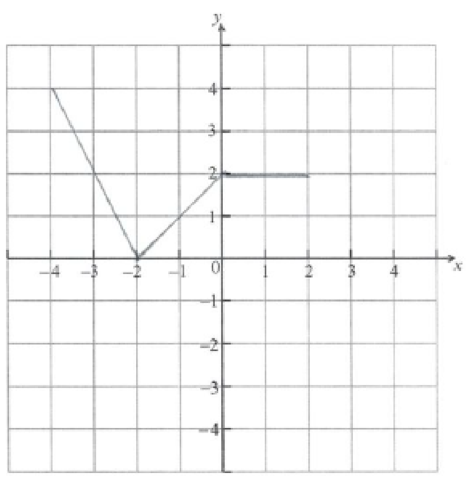
a. Let $h(x)=f(-x)$. Sketch the graph of h on the grid below. [3]



b. Let $g(x) = \frac{1}{2}f(x-1)$. The point $\mathrm{A}(3,\, 2)$ on the graph of f is transformed to the point P on the graph of g . Find the coordinates of P. [3]

Markscheme

a.



A2 N2

[2 marks]

b. evidence of appropriate approach ***(M1)***

e.g. reference to any horizontal shift and/or stretch factor, $x = 3+1$, $y = \frac{1}{2} \times 2$

P is $(4,\, 1)$ (accept $x = 4$, $y = 1$) ***A1A1 N3***

[3 marks]

Examiners report

a. Part (a) was generally solved correctly. Students had no trouble in deciding what transformation had to be done to the graph, although some confused $f(-x)$ with $-f(x)$.

b. Part (b) was generally poorly done. They could not "read" that the transformation shifted the curve 1 unit to the right and stretched it in the y-direction with a scale factor of $\frac{1}{2}$. It was often seen that the shift was interpreted, but in the opposite direction. Also, the stretch was applied to both coordinates of the point. Those candidates who answered part (a) incorrectly often had trouble on (b) as well, indicating a difficulty with transformations in general. However, there were also candidates who solved part (a) correctly but could not interpret part (b). This would indicate that it is simpler for them to plot the transformation of an entire function than to find how a particular point is transformed.

Let $f(x) = 3(x+1)^2 - 12$.

a. Show that $f(x) = 3x^2 + 6x - 9$. [2]

b(i), (ii), (iii) and (iv). For the graph of f [8]

(i) write down the coordinates of the vertex;

(ii) write down the **equation** of the axis of symmetry;

(iii) write down the y-intercept;

(iv) find both x-intercepts.

c. **Hence** sketch the graph of f . [2]

d. Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the two transformations: [3]

a stretch of scale factor t in the y-direction

followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Find $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t.

Markscheme

a. $f(x) = 3(x^2 + 2x + 1) - 12$ ***A1***

$= 3x^2 + 6x + 3 - 12$ ***A1***

$= 3x^2 + 6x - 9$ ***AG*** ***N0***

[2 marks]

b(i), (ii), (iii) and (iv). (i) vertex is $(-1,\ -12)$ ***A1A1*** ***N2***

(ii) $x = -1$ (**must** be an equation) ***A1*** ***N1***

(iii) $(0,\ -9)$ ***A1*** ***N1***

(iv) evidence of solving $f(x) = 0$ ***(M1)***

e.g. factorizing, formula,

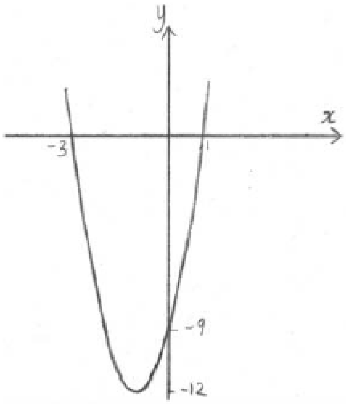
correct working ***A1***

e.g. $3(x+3)(x-1)=0$, $x=\frac{-6\pm\sqrt{36+108}}{6}$

$(-3,0)$, $(1,0)$ ***A1A1*** ***N1N1***

[8 marks]

c.



A1A1 ***N2***

Note: Award ***A1*** for a parabola opening upward, ***A1*** for vertex and intercepts in approximately correct positions.

[2 marks]

d. $\begin{pmatrix} p \\ q \end{pmatrix}=\begin{pmatrix} -1 \\ -12 \end{pmatrix}$, $t=3$ (accept $p=-1$, $q=-12$, $t=3$) ***A1A1A1*** ***N3***

[3 marks]

Examiners report

a. This problem was generally well done. The "show that" question in part (a) was done correctly by most candidates, with a few attempting to show it by working backwards, which earned no marks.

b(i), (ii) and (iv). Most candidates were able to identify the vertex but were unable to write the equation for the axis of symmetry. There was a great deal of success with the *x* and *y* intercepts.

c. Some of the sketches of the graph left much to be desired even if they were technically correct; many were v-shaped.

d. The final part was poorly done, indicating that defining a graph in terms of stretch and translation was unfamiliar to many candidates.

The equation $x^2-3x+k^2=4$ has two distinct real roots. Find the possible values of k .

Markscheme

evidence of rearranged quadratic equation (may be seen in working) ***A1***

e.g. $x^2-3x+k^2-4=0$, k^2-4

evidence of discriminant (must be seen explicitly, not in quadratic formula) ***(M1)***

e.g. b^2-4ac , $\Delta=(-3)^2-4(1)(k^2-4)$

recognizing that discriminant is greater than zero (seen anywhere, including answer) ***R1***

e.g. $b^2-4ac>0$, $9+16-4k^2>0$

correct working (accept equality) ***A1*** *e.g.* $25 - 4k^2 > 0$, $4k^2 < 25$, $k^2 = \frac{25}{4}$

both correct values (even if inequality never seen) ***(A1)***

e.g. $\pm\sqrt{\frac{25}{4}}$, ± 2.5

correct interval ***A1*** ***N3***

e.g. $-\frac{5}{2} < k < \frac{5}{2}$, $-2.5 < k < 2.5$

Note: Do not award the final mark for unfinished values, or for incorrect or reversed inequalities, including $\leq$, $k > -2.5$, $k < 2.5$.

Special cases:

If working shown, and candidates attempt to rearrange the quadratic equation to equal zero, but find an incorrect value of *c*, award ***A1M1R1A0A0A0***.

If working shown, and candidates do not rearrange the quadratic equation to equal zero, but find $c = k^2$ or $c = \pm 4$, award ***A0M1R1A0A0A0***.

[6 marks]

Examiners report

The majority of candidates who attempted to answer this question recognized the need to use the discriminant, however very few were able to answer the question successfully. The majority of candidates did not recognize that the quadratic equation must first be set equal to zero. In addition, many candidates simply set their discriminant equal to zero, instead of setting it greater than zero. Even many of the strongest candidates, who obtained the correct numerical values for *k*, were unable to write their final answers as a correct interval.

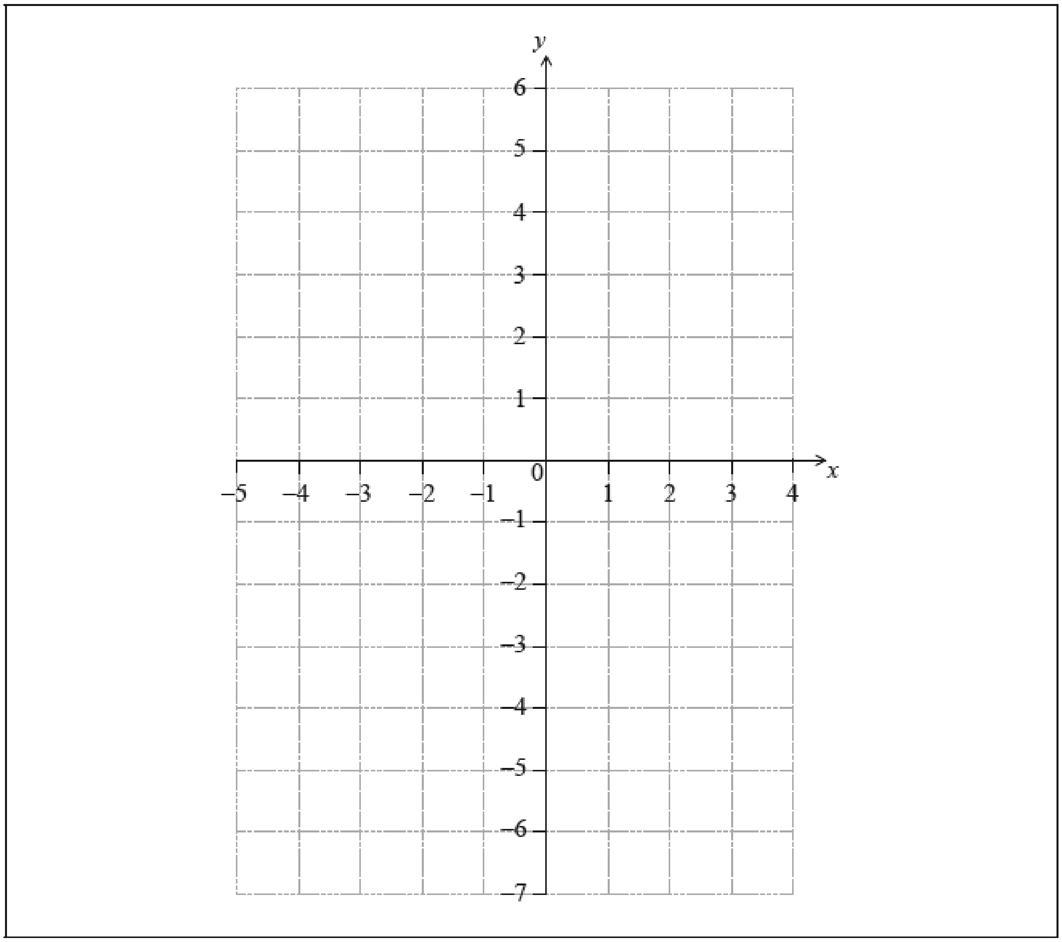
This question is a good example of candidates who reach for familiar methods, without really thinking about what the question is asking them to find. There were many candidates who attempted to solve for *x* using the quadratic formula or factoring, even though the question did not ask them to solve for *x*.

Let $f(x) = x^2 + x - 6$.

a. Write down the y-intercept of the graph of f. [1]

b. Solve $f(x) = 0$. [3]

c. On the following grid, sketch the graph of f, for $-4 \leq x \leq 3$. [3]



Markscheme

a. y-intercept is $-6,\ (0,\ -6),\ y = -6$ ***A1***

[1 mark]

b. valid attempt to solve ***(M1)***

eg $(x-2)(x+3) = 0,\ x = \frac{-1 \pm \sqrt{1+24}}{2}$, one correct answer

$x = 2,\ x = -3$ ***A1A1*** ***N3***

[3 marks]

c. ***A1A1A1***

Note: The shape must be an approximately correct concave up parabola. Only if the shape is correct, award the following:

A1 for the y-intercept in circle **and** the vertex approximately on $x = -\frac{1}{2}$, below $y = -6$,

A1 for **both** the x-intercepts in circles,

A1 for **both** end points in ovals.

[3 marks]

Total [7 marks]

Examiners report

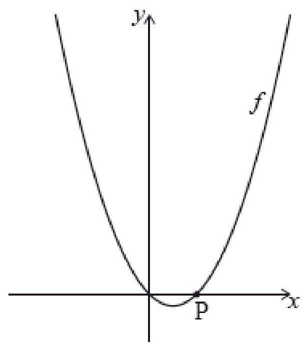
a. Parts (a) and (b) of this question were answered quite well by nearly all candidates, with only a few factoring errors in part (b).

b. Parts (a) and (b) of this question were answered quite well by nearly all candidates, with only a few factoring errors in part (b).

c. In part (c), although most candidates were familiar with the general parabolic shape of the graph, many placed the vertex at the y-intercept $(0,\ -6)$, and very few candidates considered the endpoints of the function with the given domain.

Let $f(x) = x^2 - x$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.

diagram not to scale

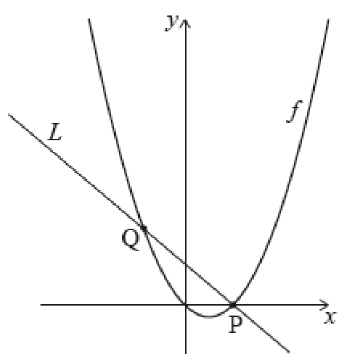


The graph of f crosses the x-axis at the origin and at the point $\mathrm{P}(1,\ 0)$.

The line *L* is the normal to the graph of *f* at P.

The line L intersects the graph of f at another point Q, as shown in the following diagram.

diagram not to scale



a. Show that $f'(1) = 1$. [3]

b. Find the equation of L in the form $y = ax + b$. [3]

c. Find the x-coordinate of Q. [4]

d. Find the area of the region enclosed by the graph of f and the line L. [6]

Markscheme

a. $f'(x) = 2x - 1$ ***A1A1***

correct substitution ***A1***

eg $2(1) - 1,\ 2 - 1$

$f'(1) = 1$ ***AG*** ***N0***

[3 marks]

b. correct approach to find the gradient of the normal ***(A1)***

eg $\frac{-1}{f'(1)},\ m_1 m_2 = -1,\ \text{slope} = -1$

attempt to substitute correct normal gradient and coordinates into equation of a line ***(M1)***

eg $y - 0 = -1(x - 1),\ 0 = -1 + b,\ b = 1,\ L = -x + 1$

$y = -x + 1$ ***A1*** ***N2***

[3 marks]

c. equating expressions ***(M1)***

eg $f(x) = L,\ -x + 1 = x^2 - x$

correct working (must involve combining terms) ***(A1)***

eg $x^2 - 1 = 0,\ x^2 = 1,\ x = 1$

$x = -1 \quad (\text{accept } Q(-1,\ 2))$ ***A2*** ***N3***

[4 marks]

d. valid approach ***(M1)***

eg $\int L - f,\ \int_{-1}^{1} (1 - x^2)\mathrm{d}x$, splitting area into triangles and integrals

correct integration ***(A1)(A1)***

eg $\left[x - \frac{x^3}{3}\right]_{-1}^{1},\ -\frac{x^3}{3} - \frac{x^2}{2} + \frac{x^2}{2} + x$

substituting **their** limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $1 - \frac{1}{3} - \left(-1 - \frac{-1}{3}\right)$

Note: Award ***M0*** for substituting into original or differentiated function.

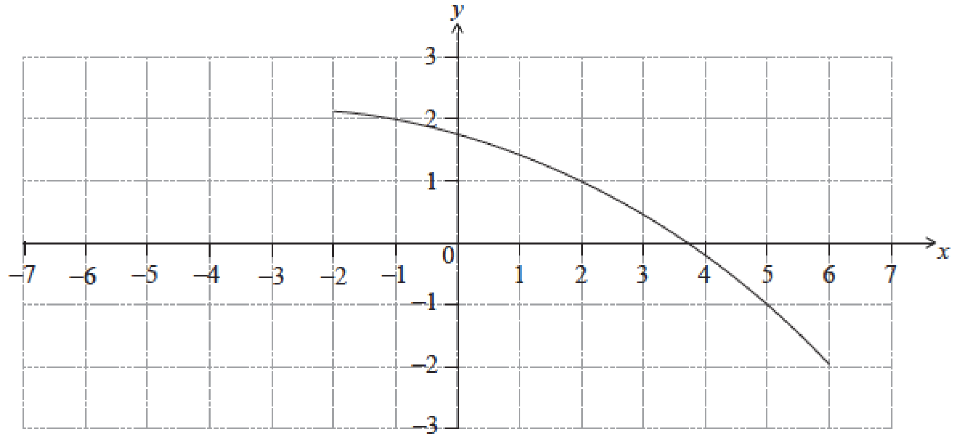
area $= \frac{4}{3}$ ***A2*** ***N3***

[6 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

The following diagram shows the graph of a function f.



a. Find $f^{-1}(-1)$. [2]

b. Find $(f \circ f)(-1)$. [3]

c. On the same diagram, sketch the graph of $y = f(-x)$. [2]

Markscheme

a. valid approach ***(M1)***

eg horizontal line on graph at -1, $f(a) = -1$, $(-1, 5)$

$f^{-1}(-1) = 5$ ***A1 N2***

[2 marks]

b. attempt to find $f(-1)$ ***(M1)***

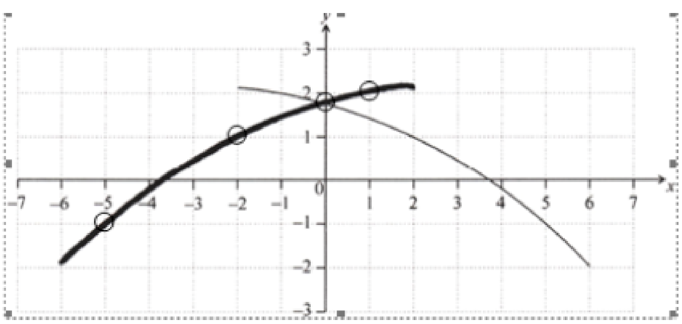
eg line on graph

$f(-1) = 2$ ***(A1)***

$(f \circ f)(-1) = 1$ ***A1 N3***

[3 marks]

c.



A1A1 N2

Note: The shape **must** be an approximately correct shape (concave down and increasing). **Only** if the shape is approximately correct, award the following for points in circles:

A1 for the y-intercept,

A1 for any **two** of these points $(-5,\ -1)$, $(-2,\ 1)$, $(1,\ 2)$.

[2 marks]

Total [7 marks]

Examiners report

a. Typically candidates were more successful in finding the composite function than the inverse. Some students tried to find the function, rather than read values from the given graph. The sketch of $f(-x)$ was often well done, with the most common error being a reflection in the *x*-axis.

b. Typically candidates were more successful in finding the composite function than the inverse. Some students tried to find the function, rather than read values from the given graph. The sketch of $f(-x)$ was often well done, with the most common error being a reflection in the *x*-axis.

c. Typically candidates were more successful in finding the composite function than the inverse. Some students tried to find the function, rather than read values from the given graph. The sketch of $f(-x)$ was often well done, with the most common error being a reflection in the *x*-axis.

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a. Find the value of $\log_2 40 - \log_2 5$. [3]

b. Find the value of $8^{\log_2 5}$. [4]

Markscheme

a. evidence of correct formula ***(M1)***

eg $\log a - \log b = \log \frac{a}{b}$, $\log\left(\frac{40}{5}\right)$, $\log 8 + \log 5 - \log 5$

Note: Ignore missing or incorrect base.

correct working ***(A1)***

eg $\log_2 8$, $2^3 = 8$

$\log_2 40 - \log_2 5 = 3$ ***A1 N2***

[3 marks]

b. attempt to write 8 as a power of 2 (seen anywhere) ***(M1)***

eg $(2^3)^{\log_2 5}$, $2^3 = 8$, 2^a

multiplying powers ***(M1)***

eg $2^{3\log_2 5}$, $a\log_2 5$

correct working ***(A1)***

eg $2^{\log_2 125}$, $\log_2 5^3$, $\left(2^{\log_2 5}\right)^3$

$8^{\log_2 5} = 125$ ***A1 N3***

[4 marks]

Examiners report

a. Many candidates readily earned marks in part (a). Some interpreted $\log_2 40 - \log_2 5$ to mean $\frac{\log_2 40}{\log_2 5}$, an error which led to no further marks. Others left the answer as $\log_2 5$ where an integer answer is expected.

b. Part (b) proved challenging for most candidates, with few recognizing that changing 8 to base 2 is a helpful move. Some made it as far as $2^{3\log_2 5}$ yet could not make that final leap to an integer.

Let $f(x) = \sqrt{x-5}$, for $x \geq 5$.

a. Find $f^{-1}(2)$. [3]

b. Let g be a function such that g^{-1} exists for all real numbers. Given that $g(30) = 3$, find $(f \circ g^{-1})(3)$. [3]

Markscheme

a. **METHOD 1**

attempt to set up equation ***(M1)***

eg $2 = \sqrt{y-5}\ , 2 = \sqrt{x-5}$

correct working ***(A1)***

eg $4 = y - 5\ , x = 2^2 + 5$

$f^{-1}(2) = 9$ ***A1 N2***

METHOD 2

interchanging x and y (seen anywhere) ***(M1)***

eg $x = \sqrt{y-5}$

correct working ***(A1)***

eg $x^2 = y - 5\ , y = x^2 + 5$

$f^{-1}(2) = 9$ ***A1 N2***

[3 marks]

b. recognizing $g^{-1}(3) = 30$ ***(M1)***

eg $f(30)$

correct working ***(A1)***

eg $(f \circ g^{-1})(3) = \sqrt{30-5}\ , \sqrt{25}$

$(f \circ g^{-1})(3) = 5$ ***A1 N2***

Note: Award ***A0*** for multiple values, *eg* ± 5 .

[3 marks]

Examiners report

a. Candidates often found an inverse function in which to substitute the value of 2. Some astute candidates set the function equal to 2 and solved for x. Occasionally a candidate misunderstood the notation as asking for a derivative, or used $\frac{1}{f(x)}$.

b. For part (b), many candidates recognized that if $g(30) = 3$ then $g^{-1}(3) = 30$, and typically completed the question successfully. Occasionally, however, a candidate incorrectly answered $\sqrt{25} = \pm 5$.

The equation $x^2 + (k+2)x + 2k = 0$ has two distinct real roots.

Find the possible values of k.

Markscheme

evidence of discriminant ***(M1)***

eg $b^2 - 4ac,\ \Delta = 0$
correct substitution into discriminant ***(A1)***
eg $(k+2)^2 - 4(2k),\ k^2 + 4k + 4 - 8k$
correct discriminant ***A1***
eg $k^2 - 4k + 4,\ (k-2)^2$
recognizing discriminant is positive ***R1***
eg $\Delta > 0,\ (k+2)^2 - 4(2k) > 0$
attempt to solve **their** quadratic in k ***(M1)***
eg factorizing, $k = \frac{4 \pm \sqrt{16-16}}{2}$
correct working ***A1***
eg $(k-2)^2 > 0,\ k = 2$, sketch of positive parabola on the x-axis
correct values ***A2*** ***N4***
eg $k \in \mathbb{R}$ and $k \neq 2,\ \mathbb{R}\backslash 2,\ ]-\infty,\ 2[\cup]2,\ \infty[$
[8 marks]

Examiners report

[N/A]

Let $f(x) = 3\ln x$ and $g(x) = \ln 5x^3$.

a. Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$. [4]

b. The graph of g is a transformation of the graph of f . Give a full geometric description of this transformation. [3]

Markscheme

a. attempt to apply rules of logarithms ***(M1)***

e.g. $\ln a^b = b\ln a$, $\ln ab = \ln a + \ln b$

correct application of $\ln a^b = b\ln a$ (seen anywhere) ***A1***

e.g. $3\ln x = \ln x^3$

correct application of $\ln ab = \ln a + \ln b$ (seen anywhere) ***A1***

e.g. $\ln 5x^3 = \ln 5 + \ln x^3$

so $\ln 5x^3 = \ln 5 + 3\ln x$

$g(x) = f(x) + \ln 5$ (accept $g(x) = 3\ln x + \ln 5$) ***A1*** ***N1***

[4 marks]

b. transformation with correct name, direction, and value ***A3*** e.g. translation by $\begin{pmatrix} 0 \\ \ln 5 \end{pmatrix}$, shift up by $\ln 5$, vertical translation of $\ln 5$

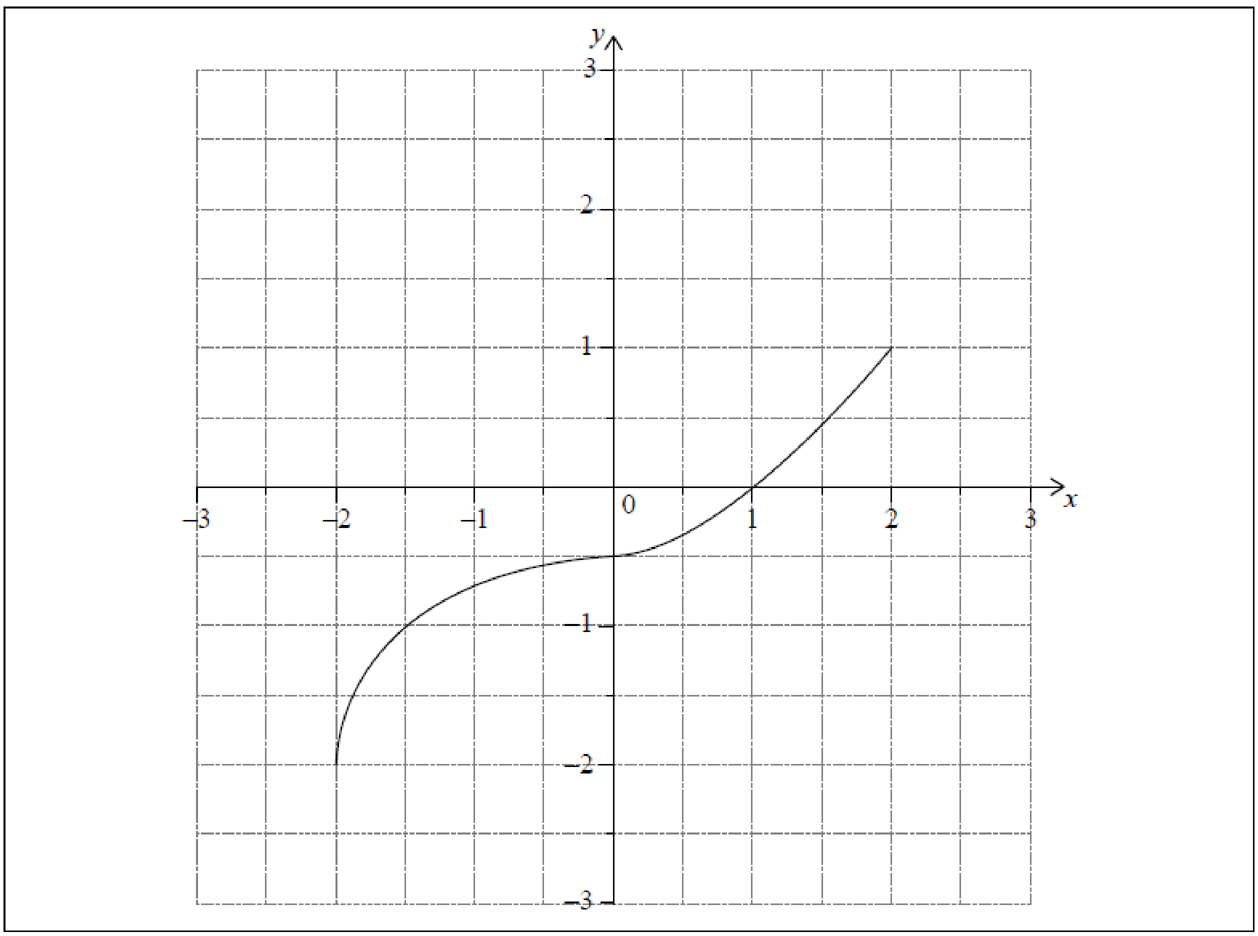
[3 marks]

Examiners report

a. This question was very poorly done by the majority of candidates. While candidates seemed to have a vague idea of how to apply the rules of logarithms in part (a), very few did so successfully. The most common error in part (a) was to begin incorrectly with $\ln 5x^3 = 3\ln 5x$. This error was often followed by other errors.

b. In part (b), very few candidates were able to describe the transformation as a vertical translation (or shift). Many candidates attempted to describe numerous incorrect transformations, and some left part (b) entirely blank.

Consider a function *f*(*x*) , for −2 ≤ *x* ≤ 2 . The following diagram shows the graph of *f*.



a.i. Write down the value of *f*(0). [1]

a.ii. Write down the value of $f^{-1}(1)$. [1]

b. Write down the range of f^{-1}. [1]

c. On the grid above, sketch the graph of f^{-1}. [4]

Markscheme

a.i. $f(0) = -\frac{1}{2}$ ***A1 N1***

[1 mark]

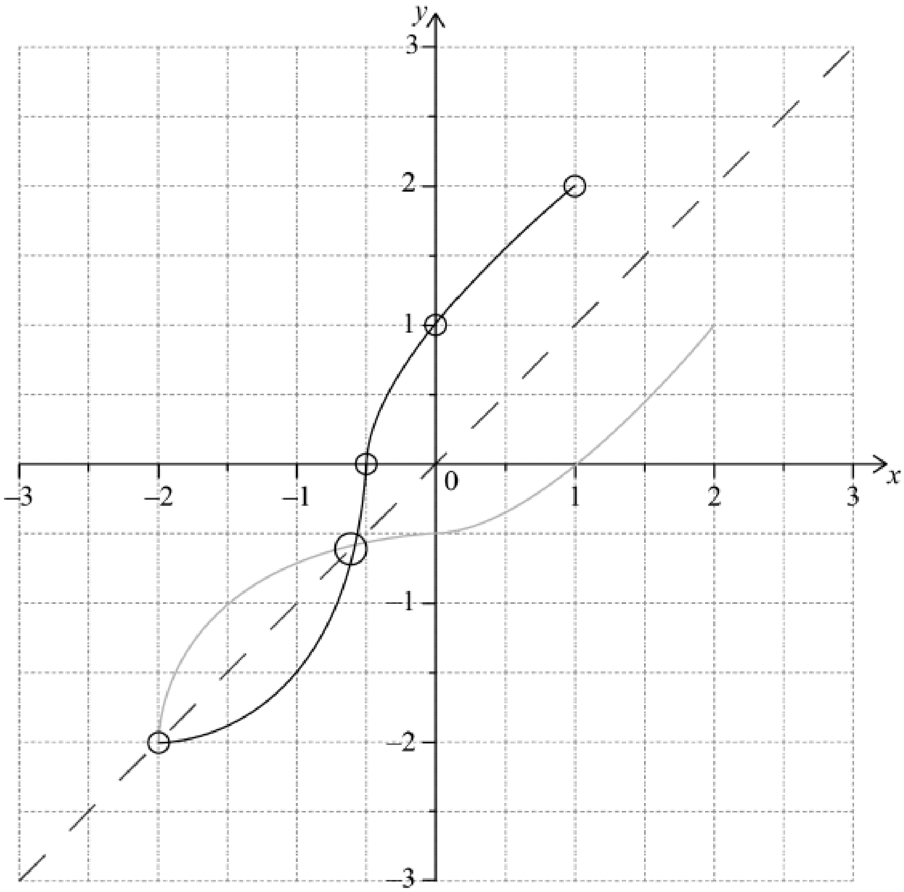
a.ii $f^{-1}(1) = 2$ ***A1 N1***

[1 mark]

b. $-2 \le y \le 2$, $y \in [-2, 2]$ (accept $-2 \le x \le 2$) ***A1 N1***

[1 mark]

c.



A1A1A1A1 N4

Note: Award ***A1*** for evidence of approximately correct reflection in $y = x$ with correct curvature.

($y = x$ does not need to be explicitly seen)

Only if this mark is awarded, award marks as follows:

A1 for both correct invariant points in circles,

A1 for the three other points in circles,

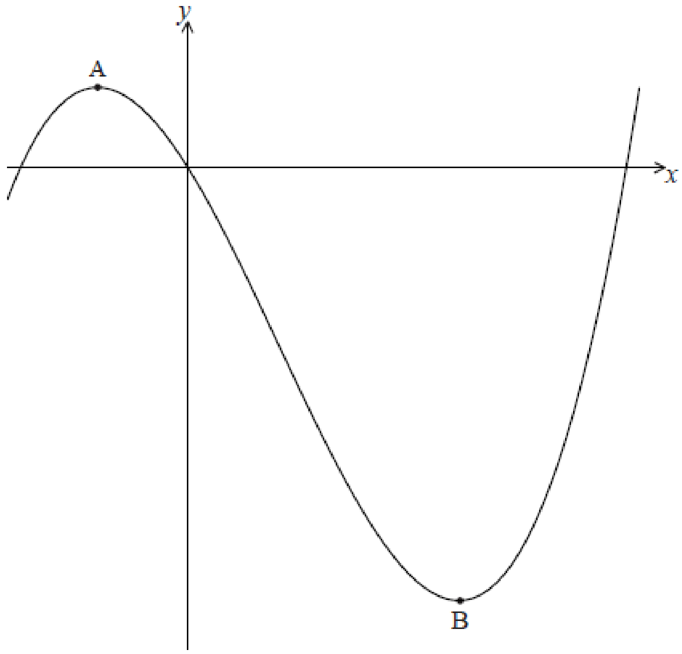
A1 for correct domain.

[4 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]
c. [N/A]

Let $f(x) = \frac{1}{2}x^3 - x^2 - 3x$. Part of the graph of *f* is shown below.



There is a maximum point at A and a minimum point at B$(3, -9)$.

a. Find the coordinates of A. [8]

b(i), (ii) and (iii). Write down the coordinates of [6]

(i) the image of B after reflection in the *y*-axis;

(ii) the image of B after translation by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$;

(iii) the image of B after reflection in the *x*-axis followed by a horizontal stretch with scale factor $\frac{1}{2}$.

Markscheme

a. $f(x) = x^2 - 2x - 3$ ***A1A1A1***

evidence of solving $f'(x) = 0$ ***(M1)***

e.g. $x^2 - 2x - 3 = 0$

evidence of correct working ***A1***

e.g. $(x + 1)(x - 3)$, $\frac{2 \pm \sqrt{16}}{2}$

$x = -1$ (ignore $x = 3$) ***(A1)***

evidence of substituting **their** negative *x*-value into $f(x)$ ***(M1)***

e.g. $\frac{1}{3}(-1)^3 - (-1)^2 - 3(-1)$, $-\frac{1}{3} - 1 + 3$

$y = \frac{5}{3}$ ***A1***

coordinates are $\left(-1, \frac{5}{3}\right)$ ***N3***

[8 marks]

b(i), (ii) and (iii). $(-3, -9)$ ***A1*** ***N1***

(ii) $(1,\ -4)$ ***A1A1*** ***N2***

(iii) reflection gives $(3, 9)$ ***(A1)***

stretch gives $\left(\frac{3}{2}, 9\right)$ ***A1A1*** ***N3***

[6 marks]

Examiners report

a. A majority of candidates answered part (a) completely.

b(i), (ii) and (iii). Candidates were generally successful in finding images after single transformations in part (b). Common incorrect answers for (biii) included $\left(\frac{3}{2}, \frac{9}{2}\right)$, $(6, 9)$ and $(6, 18)$, demonstrating difficulty with images from horizontal stretches.

Let $f'(x) = \frac{6-2x}{6x-x^2}$, for $0 < x < 6$.

The graph of f has a maximum point at P.

The y-coordinate of P is $\ln 27$.

a. Find the x-coordinate of P. [3]

b. Find $f(x)$, expressing your answer as a single logarithm. [8]

c. The graph of f is transformed by a vertical stretch with scale factor $\frac{1}{\ln 3}$. The image of P under this transformation has coordinates $(a,\ b)$. [[N/A]]

Find the value of a and of b, where $a,\ b \in \mathbb{N}$.

Markscheme

a. recognizing $f'(x) = 0$ ***(M1)***

correct working ***(A1)***

eg $6 - 2x = 0$

$x = 3$ ***A1*** ***N2***

[3 marks]

b. evidence of integration ***(M1)***

eg $\int f',\ \int \frac{6-2x}{6x-x^2}\mathrm{d}x$

using substitution ***(A1)***

eg $\int \frac{1}{u}\mathrm{d}u$ where $u = 6x - x^2$

correct integral ***A1***

eg $\ln(u) + c,\ \ln(6x - x^2)$

substituting $(3,\ \ln 27)$ into **their** integrated expression (must have c) ***(M1)***

eg $\ln(6 \times 3 - 3^2) + c = \ln 27,\ \ln(18 - 9) + \ln k = \ln 27$

correct working ***(A1)***

eg $c = \ln 27 - \ln 9$

EITHER

$c = \ln 3$ ***(A1)***

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x) = \ln(6x - x^2) + \ln 3$ ***A1 N4***

OR

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x) = \ln(6x - x^2) + \ln 27 - \ln 9$

correct use of a log law ***(A1)***

eg $f(x) = \ln(6x - x^2) + \ln\left(\frac{27}{9}\right),\ f(x) = \ln\big(27(6x - x^2)\big) - \ln 9$

$f(x) = \ln\big(3(6x - x^2)\big)$ ***A1 N4***

[8 marks]

c. $a = 3$ ***A1 N1***

correct working ***A1***

eg $\frac{\ln 27}{\ln 3}$

correct use of log law ***(A1)***

eg $\frac{3\ln 3}{\ln 3},\ \log_3 27$

$b = 3$ ***A1 N2***

[4 marks]

Examiners report

a. Part a) was well answered.

b. In part b) most candidates realised that integration was required but fewer recognised the need to use integration by substitution. Quite a number of candidates who integrated correctly omitted finding the constant of integration.

c. In part c) many candidates showed good understanding of transformations and could apply them correctly, however, correct use of the laws of logarithms was challenging for many. In particular, a common error was $\frac{\ln 27}{\ln 3} = \ln 9$.

Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots.

a. Find the value of k . [5]

b. The line $y = p$ intersects the graph of f . Find all possible values of p . [2]

Markscheme

a. valid approach *(M1)*

e.g. $b^2 - 4ac$, $\Delta = 0$, $(-4k)^2 - 4(2k)(1)$

correct equation *A1*

e.g. $(-4k)^2 - 4(2k)(1) = 0$, $16k^2 = 8k$, $2k^2 - k = 0$

correct manipulation *A1*

e.g. $8k(2k-1)$, $\frac{8\pm\sqrt{64}}{32}$

$k = \frac{1}{2}$ *A2* *N3*

[5 marks]

b. recognizing vertex is on the x-axis ***M1***

e.g. (1, 0) , sketch of parabola opening upward from the x-axis

$p \geq 0$ ***A1*** ***N1***

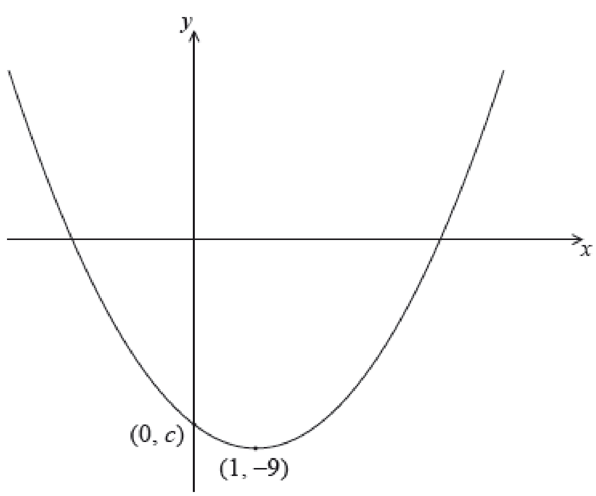
[2 marks]

Examiners report

a. Those who knew to set the discriminant to zero had little trouble completing part (a). Some knew that having two equal roots means the factors must be the same, and thus surmised that $k = \frac{1}{2}$ will achieve $(x-1)(x-1)$. This is a valid approach, provided the reasoning is completely communicated. Many candidates set $f = 0$ and used the quadratic formula, which misses the approach entirely.

b. Part (b) proved challenging for most, and was often left blank. Those who considered a graphical interpretation and sketched the parabola found greater success.

The following diagram shows part of the graph of a quadratic function f.



The vertex is at $(1, \ -9)$, and the graph crosses the y-axis at the point $(0, \ c)$.

The function can be written in the form $f(x) = (x-h)^2 + k$.

a. Write down the value of h and of k. [2]

b. Find the value of c. [2]

c. Let $g(x) = -(x-3)^2 + 1$. The graph of g is obtained by a reflection of the graph of f in the x-axis, followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$. [5]

Find the value of p and of q.

d. Find the *x*-coordinates of the points of intersection of the graphs of f and g. [7]

Markscheme

a. $h = 1,\ k = -9 \quad \left(\text{accept } (x-1)^2 - 9\right)$ ***A1A1*** ***N2***

[2 marks]

b. **METHOD 1**

attempt to substitute $x = 0$ into **their** quadratic function ***(M1)***

eg $f(0),\ (0-1)^2 - 9$

$c = -8$ ***A1*** ***N2***

METHOD 2

attempt to expand **their** quadratic function ***(M1)***

eg $x^2 - 2x + 1 - 9,\ x^2 - 2x - 8$

$c = -8$ ***A1*** ***N2***

[2 marks]

c. evidence of correct reflection ***A1***

eg $-\left((x-1)^2 - 9\right)$, vertex at $(1,\ 9)$, *y*-intercept at $(0,\ 8)$

valid attempt to find horizontal shift ***(M1)***

eg $1 + p = 3,\ 1 \to 3$

$p = 2$ ***A1*** ***N2***

valid attempt to find vertical shift ***(M1)***

eg $9 + q = 1,\ 9 \to 1,\ \ -9 + q = 1$

$q = -8$ ***A1*** ***N2***

Notes: An error in finding the reflection may still allow the correct values of p and q to be found, as the error may not affect subsequent working. In this case, award ***A0*** for the reflection, ***M1A1*** for $p = 2$, and ***M1A1*** for $q = -8$.

If no working shown, award ***N0*** for $q = 10$.

[5 marks]

d. valid approach (check ***FT*** from (a)) ***M1***

eg $f(x) = g(x),\ (x-1)^2 - 9 = -(x-3)^2 + 1$

correct expansion of both binomials ***(A1)***

eg $x^2 - 2x + 1,\ x^2 - 6x + 9$

correct working ***(A1)***

eg $x^2 - 2x - 8 = -x^2 + 6x - 8$

correct equation ***(A1)***

eg $2x^2 - 8x = 0,\ 2x^2 = 8x$

correct working ***(A1)***

eg $2x(x-4)=0$

$x=0,\ x=4$ ***A1A1*** ***N3***

[7 marks]

Total [16 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

The following table shows the probability distribution of a discrete random variable A, in terms of an angle θ.

a	1	2
$\mathrm{P}(A=a)$	$\cos\theta$	$2\cos 2\theta$

a. Show that $\cos\theta=\frac{3}{4}$. [6]

b. Given that $\tan\theta>0$, find $\tan\theta$. [3]

c. Let $y=\frac{1}{\cos x}$, for $0<x<\frac{\pi}{2}$. The graph of $y$between $x=\theta$ and $x=\frac{\pi}{4}$ is rotated 360° about the x-axis. Find the volume of the solid formed. [6]

Markscheme

a. evidence of summing to 1 ***(M1)***

eg $\sum p=1$

correct equation ***A1***

eg $\cos\theta+2\cos 2\theta=1$

correct equation in $\cos\theta$ ***A1***

eg $\cos\theta+2(2\cos^2\theta-1)=1,\ 4\cos^2\theta+\cos\theta-3=0$

evidence of valid approach to solve quadratic ***(M1)***

eg factorizing equation set equal to 0, $\frac{-1\pm\sqrt{1-4\times4\times(-3)}}{8}$

correct working, clearly leading to required answer ***A1***

eg $(4\cos\theta-3)(\cos\theta+1)$, $\frac{-1\pm7}{8}$

correct reason for rejecting $\cos\theta\neq-1$ ***R1***

eg $\cos\theta$ is a probability (value must lie between 0 and 1), $\cos\theta>0$

Note: Award ***R0*** for $\cos\theta\neq-1$ without a reason.

$\cos\theta=\frac{3}{4}$ ***AG N0***

b. valid approach ***(M1)***

eg sketch of right triangle with sides 3 and 4, $\sin^2 x + \cos^2 x = 1$

correct working

(A1)

eg missing side $= \sqrt{7}$, $\frac{\frac{\sqrt{7}}{4}}{\frac{3}{4}}$

$\tan\theta = \frac{\sqrt{7}}{3}$ ***A1*** ***N2***

[3 marks]

c. attempt to substitute either limits or the function into formula involving f^2 ***(M1)***

eg $\pi\int_\theta^{\frac{\pi}{4}} f^2$, $\int\left(\frac{1}{\cos x}\right)^2$

correct substitution of both limits and function ***(A1)***

eg $\pi\int_\theta^{\frac{\pi}{4}}\left(\frac{1}{\cos x}\right)^2 \mathrm{d}x$

correct integration ***(A1)***

eg $\tan x$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

eg $\tan\frac{\pi}{4} - \tan\theta$

Note: Award ***M0*** if they substitute into original or differentiated function.

$\tan\frac{\pi}{4} = 1$ ***(A1)***

eg $1 - \tan\theta$

$V = \pi - \frac{\pi\sqrt{7}}{3}$ ***A1*** ***N3***

[6 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Let $f(x) = x^2$ and $g(x) = 2x - 3$.

a. Find $g^{-1}(x)$. [2]

b. Find $(f\circ g)(4)$. [3]

Markscheme

a. for interchanging x and y (may be done later) ***(M1)***

e.g. $x = 2y - 3$

$g^{-1}(x) = \frac{x+3}{2}$ (accept $y = \frac{x+3}{2}, \frac{x+3}{2}$) ***A1*** ***N2***

[2 marks]

b. **METHOD 1**

$g(4) = 5$ ***(A1)***

evidence of composition of functions ***(M1)***

$f(5) = 25$ ***A1 N3***

METHOD 2

$f \circ g(x) = (2x - 3)^2$ ***(M1)***

$f \circ g(4) = (2 \times 4 - 3)^2$ ***(A1)***

$= 25$ ***A1*** ***N3***

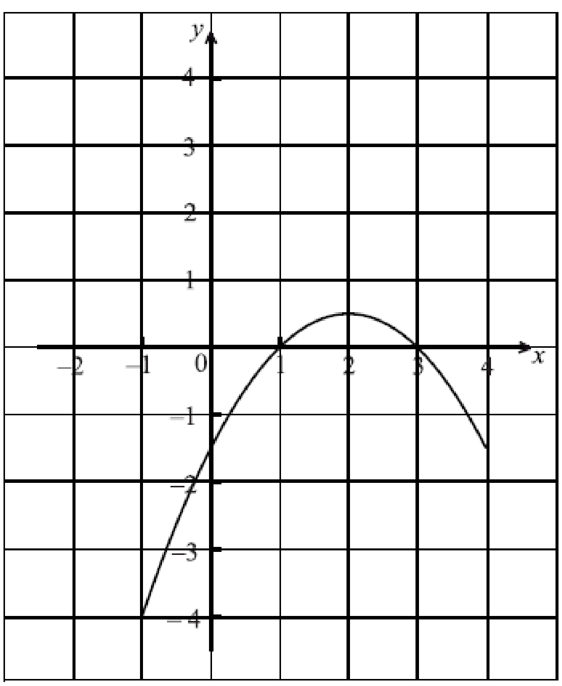
[3 marks]

Examiners report

a. Many candidates performed successfully in finding the inverse function, as well as the composite at a specified value of *x*.

b. Many candidates performed successfully in finding the inverse function, as well as the composite at a specified value of *x*. Some candidates made arithmetical errors especially if they expanded the binomial before substituting $x = 4$.

Part of the graph of a function *f* is shown in the diagram below.



a. On the same diagram sketch the graph of $y = -f(x)$. [2]

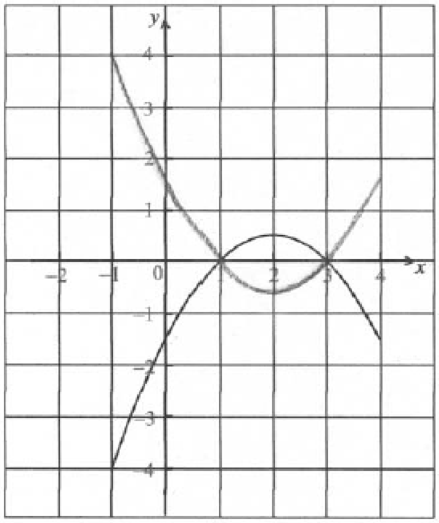
b(i) and (ii). Let $g(x) = f(x+3)$. [4]

(i) Find $g(-3)$.

(ii) Describe fully the transformation that maps the graph of *f* to the graph of *g*.

Markscheme

a.



M1A1 ***N2***

Note: Award ***M1*** for evidence of reflection in *x*-axis, ***A1*** for correct vertex **and** all intercepts approximately correct.

b(i) and (ii). (i) $g(-3) = f(0)$ ***(A1)***

$f(0) = -1.5$ ***A1*** ***N2***

(ii) translation (accept shift, slide, etc.) of $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ ***A1A1*** ***N2***

[4 marks]

Examiners report

a. This question was reasonably well done. Many recognized the graph of $-f(x)$ as a reflection in a horizontal line, but fewer recognized the *x*-axis as the mirror line.

b(i) and (ii). A fair number gave $g(-3) = f(0)$, but did not carry through to $f(0) = -1.5$. The majority of candidates recognized that moving the graph of $f(x)$ by 3 units to the left results in the graph of $g(x)$, but the language used to describe the transformation was often far from precise mathematically.

Consider the equation $x^2 + (k-1)x + 1 = 0$, where *k* is a real number.

Find the values of *k* for which the equation has two **equal** real solutions.

Markscheme

METHOD 1

evidence of valid approach ***(M1)***

e.g. $b^2 - 4ac$, quadratic formula

correct substitution into $b^2 - 4ac$ (may be seen in formula) ***(A1)*** e.g. $(k-1)^2-4\times1\times1$, $(k-1)^2-4$, k^2-2k-3

setting **their** discriminant equal to zero ***M1***

e.g. $\Delta=0, (k-1)^2-4=0$

attempt to solve the quadratic ***(M1)***

e.g. $(k-1)^2=4$, factorizing

correct working ***A1***

e.g. $(k-1)=\pm2$, $(k-3)(k+1)$

$k=-1$, $k=3$ (do not accept inequalities) ***A1A1*** ***N2***

[7 marks]

METHOD 2

recognizing perfect square ***(M1)***

e.g. $(x+1)^2=0$, $(x-1)^2$

correct expansion ***(A1)(A1)***

e.g. $x^2+2x+1=0$, x^2-2x+1

equating coefficients of *x* ***A1A1***

e.g. $k-1=-2$, $k-1=2$

$k=-1$, $k=3$ ***A1A1*** ***N2***

[7 marks]

Examiners report

Most candidates approached this question correctly by using the discriminant, and many were successful in finding both of the required values of *k*. There did seem to be some confusion about the expression "two **equal** real solutions", as some candidates approached the question as though the equation had two distinct real roots, using $b^2-4ac>0$, rather than $b^2-4ac=0$.

There were also a good number who recognized that the quadratic must be a perfect square, although many who used this method found only one of the two possible values of *k*. In addition, there were many unsuccessful candidates who tried to use the entire quadratic formula as though they were solving for *x*, without ever seeming to realize the significance of the discriminant.

---

a. Write the expression $3\ln 2-\ln 4$ in the form $\ln k$, where $k\in\mathbb{Z}$. [3]

b. Hence or otherwise, solve $3\ln 2-\ln 4=-\ln x$. [3]

Markscheme

a. correct application of $\ln a^b=b\ln a$ (seen anywhere) ***(A1)***

eg $\ln 4=2\ln 2,\ 3\ln 2=\ln 2^3,\ 3\log 2=\log 8$

correct working ***(A1)***

eg $3\ln 2-2\ln 2,\ \ln 8-\ln 4$

$\ln 2$ (accept $k=2$) ***A1*** ***N2***

[3 marks]

b. **METHOD 1**

attempt to substitute **their** answer into the equation ***(M1)***

eg $\ln 2 = -\ln x$

correct application of a log rule ***(A1)***

eg $\ln \frac{1}{x},\ \ln \frac{1}{2} = \ln x,\ \ln 2 + \ln x = \ln 2x \quad (= 0)$

$x = \frac{1}{2}$ ***A1 N2***

METHOD 2

attempt to rearrange equation, with $3\ln 2$ written as $\ln 2^3$ or $\ln 8$ ***(M1)***

eg $\ln x = \ln 4 - \ln 2^3,\ \ln 8 + \ln x = \ln 4,\ \ln 2^3 = \ln 4 - \ln x$

correct working applying $\ln a \pm \ln b$ ***(A1)***

eg $\frac{4}{8},\ 8x = 4,\ \ln 2^3 = \ln \frac{4}{x}$

$x = \frac{1}{2}$ ***A1 N2***

[3 marks]

Total [6 marks]

Examiners report

a. Part (a) was answered correctly by a large number of candidates, though there were quite a few who applied the rules of logarithms in the wrong order.

b. In part (b), many candidates knew to set their answer from part (a) equal to $-\ln x$, but then a good number incorrectly said that $\ln 2 = -\ln x$ led to $2 = -x$.

Let $f(x) = px^2 + qx - 4p$, where p ≠ 0. Find Find the number of roots for the equation $f(x) = 0$.

Justify your answer.

Markscheme

METHOD 1

evidence of discriminant ***(M1)***
eg $b^2 - 4ac,\ \Delta$

correct substitution into discriminant ***(A1)***
eg $q^2 - 4p(-4p)$

correct discriminant ***A1***
eg $q^2 + 16p^2$

$16p^2 > 0 \quad (\text{accept } p^2 > 0)$ ***A1***

$q^2 \geqslant 0 \quad (\text{do not accept } q^2 > 0)$ ***A1***

$q^2 + 16p^2 > 0$ ***A1***

f has 2 roots ***A1 N0***

METHOD 2

y-intercept = –4*p* (seen anywhere)   ***A1***

if *p* is positive, then the *y*-intercept will be negative   ***A1***

an upward-opening parabola with a negative *y*-intercept   ***R1***
eg sketch that must indicate $p > 0$.

if *p* is negative, then the y-intercept will be positive   ***A1***

a downward-opening parabola with a positive y-intercept   ***R1***
eg sketch that must indicate $p > 0$.

f has 2 roots   ***A2 N0***

[7 marks]

Examiners report

[N/A]

Let $f(x) = ax^2 - 4x - c$. A horizontal line, L, intersects the graph of f at $x = -1$ and $x = 3$.

a.i. The equation of the axis of symmetry is $x = p$. Find p. [2]

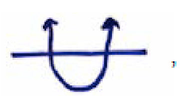
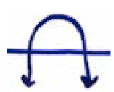
a.ii. Hence, show that $a = 2$. [2]

b. The equation of L is $y = 5$. Find the value of c. [3]

Markscheme

a.i. **METHOD 1** (using symmetry to find *p*)

valid approach   ***(M1)***

eg $\frac{-1+3}{2}$,  , 

$p = 1$ ***A1 N2***

Note: Award no marks if they work backwards by substituting $a = 2$ into $-\frac{b}{2a}$ to find *p*.

Do not accept $p = \frac{2}{a}$.

METHOD 2 (calculating *a* first)
(i) & (ii) valid approach to calculate *a*   ***M1***

eg $a + 4 - c = a(3^2) - 4(3) - c$, $f(-1) = f(3)$

correct working   ***A1***

eg $8a = 16$

$a = 2$ ***AG N0***

valid approach to find *p*   ***(M1)***

eg $-\frac{b}{2a}$, $\frac{4}{2(2)}$

$p = 1$ ***A1 N2***

[2 marks]

a.ii. **METHOD 1**

valid approach ***M1***

eg $-\frac{b}{2a}$, $\frac{4}{2a}$ (might be seen in (i)), $f'(1) = 0$

correct equation ***A1***

eg $\frac{4}{2a} = 1$, $2a(1) - 4 = 0$

$a = 2$ ***AG N0***

METHOD 2 (calculating *a* first)
(i) & (ii) valid approach to calculate a ***M1***

eg $a + 4 - c = a(3^2) - 4(3) - c$, $f(-1) = f(3)$

correct working ***A1***

eg $8a = 16$

$a = 2$ ***AG N0***

[2 marks]

b. valid approach ***(M1)***

eg $f(-1) = 5$, $f(3) = 5$

correct working ***(A1)***
eg $2 + 4 - c = 5$, $18 - 12 - c = 5$

$c = 1$ ***A1 N2***

[3 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]

Let $f(x) = \mathrm{e}^{x+3}$.

a. (i) Show that $f^{-1}(x) = \ln x - 3$. [3]

(ii) Write down the domain of f^{-1} .

b. Solve the equation $f^{-1}(x) = \ln \frac{1}{x}$. [4]

Markscheme

a. (i) interchanging *x* and *y* (seen anywhere) ***M1***

e.g. $x = \mathrm{e}^{y+3}$

correct manipulation ***A1***

e.g. $\ln x = y + 3$, $\ln y = x + 3$

$f^{-1}(x) = \ln x - 3$ ***AG*** ***N0***

(ii) $x > 0$ ***A1*** ***N1***

[3 marks]

b. collecting like terms; using laws of logs ***(A1)(A1)***

e.g. $\ln x - \ln\left(\frac{1}{x}\right) = 3$, $\ln x + \ln x = 3$, $\ln\left(\frac{x}{\frac{1}{x}}\right) = 3$, $\ln x^2 = 3$

simplify ***(A1)***

e.g. $\ln x = \frac{3}{2}$, $x^2 = \mathrm{e}^3$

$x = \mathrm{e}^{\frac{3}{2}} \left(= \sqrt{\mathrm{e}^3}\right)$ ***A1*** ***N2***

[4 marks]

Examiners report

a. Many candidates interchanged the x and y to find the inverse function, but very few could write down the correct domain of the inverse, often giving $x \geq 0$, $x > 3$ and "all real numbers" as responses.

b. Where students attempted to solve the equation in (b), most treated $\ln x - 3$ as $\ln(x - 3)$ and created an incorrect equation from the outset. The few who applied laws of logarithms often carried the algebra through to completion.

Three consecutive terms of a geometric sequence are $x - 3$, 6 and $x + 2$.

Find the possible values of x.

Markscheme

METHOD 1

valid approach ***(M1)***

eg $r = \frac{6}{x-3},\ (x-3) \times r = 6,\ (x-3)r^2 = x + 2$

correct equation in terms of x only ***A1***

eg $\frac{6}{x-3} = \frac{x+2}{6},\ (x-3)(x+2) = 6^2,\ 36 = x^2 - x - 6$

correct working ***(A1)***

eg $x^2 - x - 42,\ x^2 - x = 42$

valid attempt to solve **their** quadratic equation ***(M1)***

eg factorizing, formula, completing the square

evidence of correct working ***(A1)***

eg $(x-7)(x+6),\ \frac{1 \pm \sqrt{169}}{2}$

$x = 7,\ x = -6$ ***A1*** ***N4***

METHOD 2 (finding *r* first)

valid approach ***(M1)***

eg $r = \frac{6}{x-3},\ 6r = x+2,\ (x-3)r^2 = x+2$

correct equation in terms of r only ***A1***

eg $\frac{6}{r} + 3 = 6r - 2,\ 6 + 3r = 6r^2 - 2r,\ 6r^2 - 5r - 6 = 0$

evidence of correct working ***(A1)***

eg $(3r+2)(2r-3),\ \frac{5 \pm \sqrt{25+144}}{12}$

$r = -\frac{2}{3},\ r = \frac{3}{2}$ ***A1***

substituting their values of r to find x ***(M1)***

eg $(x-3)\left(\frac{2}{3}\right) = 6,\ x = 6\left(\frac{3}{2}\right) - 2$

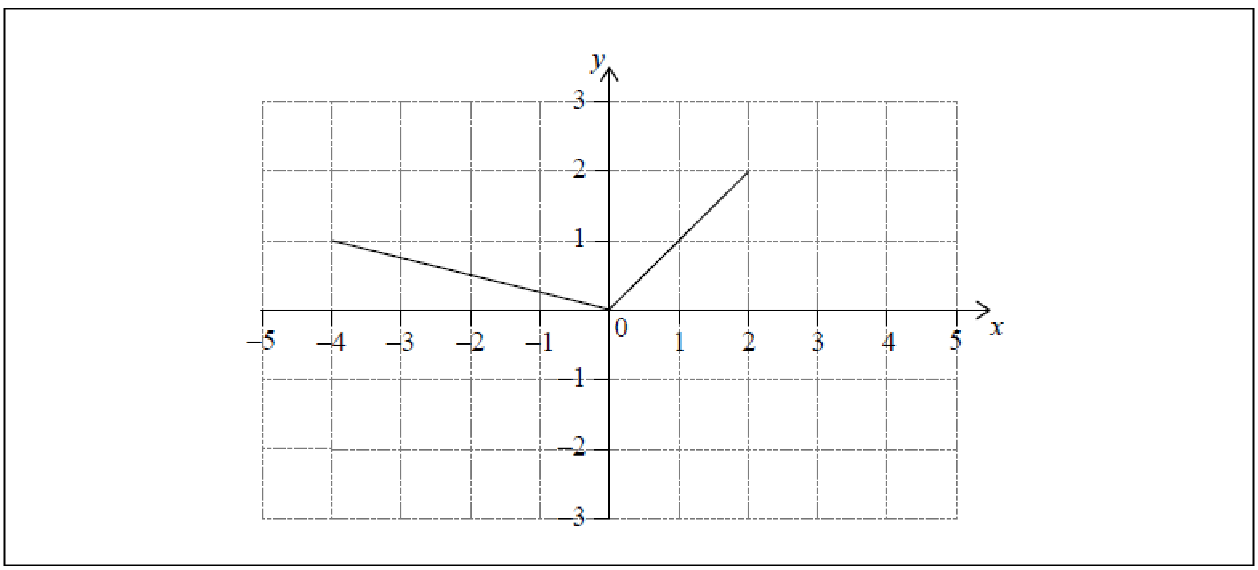
$x = 7,\ x = -6$ ***A1*** ***N4***

[6 marks]

Examiners report

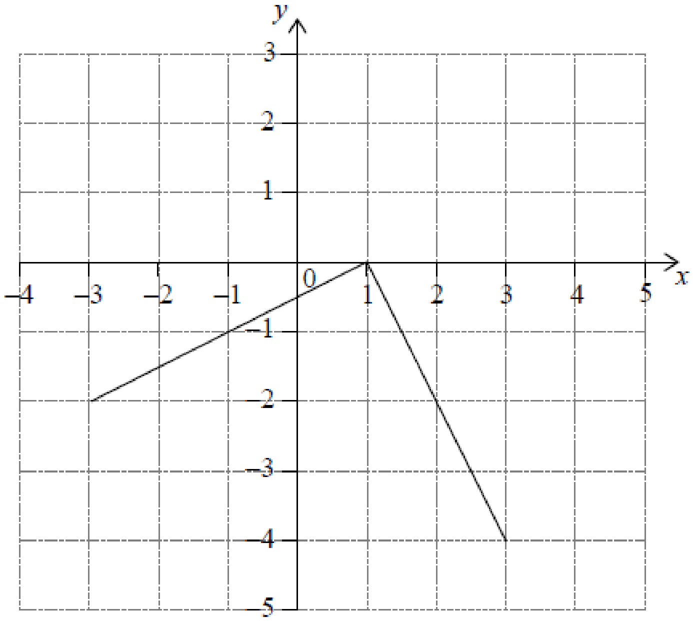
Nearly all candidates attempted to set up an expression, or pair of expressions, for the common ratio of the geometric sequence. When done correctly, these expressions led to a quadratic equation which was solved correctly by many candidates.

The following diagram shows the graph of a function f, for $-4 \le x \le 2$.



a. On the same axes, sketch the graph of $f(-x)$. [2]

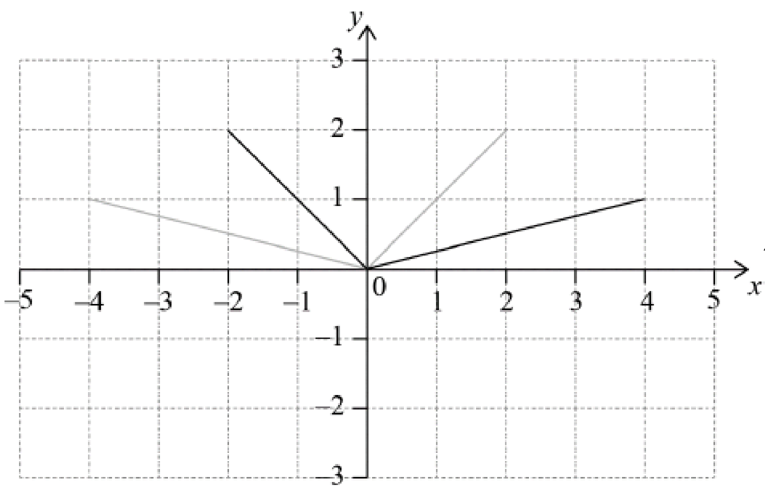
b. Another function, g, can be written in the form $g(x) = a \times f(x+b)$. The following diagram shows the graph of g. [4]



Write down the value of *a* and of *b*.

Markscheme

a.



A2 N2

[2 marks]

b. recognizing horizontal shift/translation of 1 unit ***(M1)***

eg $b = 1$, moved 1 right

recognizing vertical stretch/dilation with scale factor 2 ***(M1)***

eg $a = 2$, $y \times (-2)$

$a = -2$, $b = -1$ ***A1A1 N2N2***

[4 marks]

Examiners report

a. [N/A]

b. [N/A]

Let $f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for $0 \le x \le \pi$.

Let g be a quadratic function such that $g(0) = 5$. The line $x = 2$ is the axis of symmetry of the graph of g .

The function g can be expressed in the form $g(x) = a(x-h)^2 + 3$.

a. Find $f'(x)$. [3]

b. Find $g(4)$. [3]

c. (i) Write down the value of h . [4]

(ii) Find the value of a .

d. Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g . [6]

Markscheme

a. $f'(x) = \cos x + x - 2$ ***A1A1A1*** ***N3***

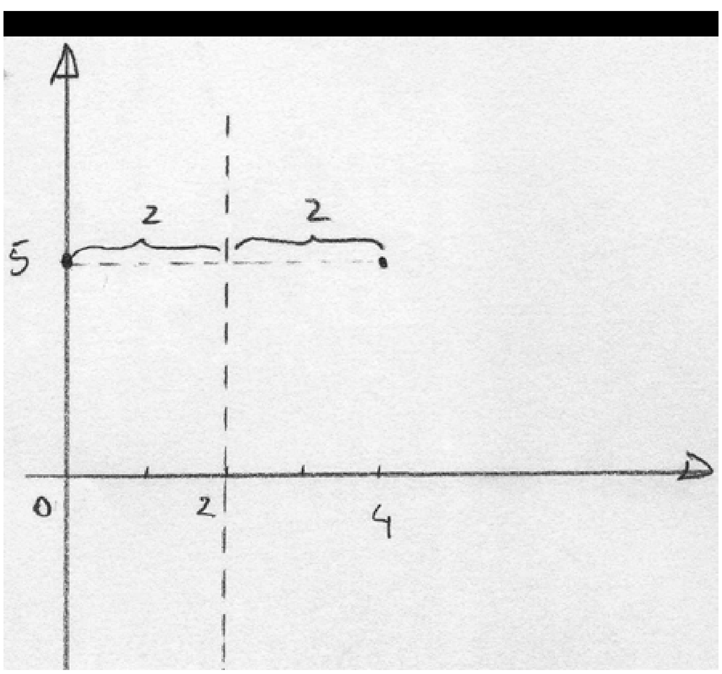
Note: Award ***A1*** for each term.

[3 marks]

b. recognizing $g(0) = 5$ gives the point (0, 5) ***(R1)***

recognize symmetry ***(M1)***

eg vertex, sketch



$g(4) = 5$ ***A1*** ***N3***

[3 marks]

c. (i) $h = 2$ ***A1 N1***

(ii) substituting into $g(x) = a(x-2)^2 + 3$ (not the vertex) ***(M1)***

eg $5 = a(0-2)^2 + 3$, $5 = a(4-2)^2 + 3$

working towards solution ***(A1)***

eg $5 = 4a + 3$, $4a = 2$

$a = \frac{1}{2}$ ***A1 N2***

[4 marks]

d. $g(x) = \frac{1}{2}(x-2)^2 + 3 = \frac{1}{2}x^2 - 2x + 5$

correct derivative of g ***A1A1***

eg $2 \times \frac{1}{2}(x-2)$, $x - 2$

evidence of equating both derivatives ***(M1)***

eg $f' = g'$

correct equation ***(A1)***

eg $\cos x + x - 2 = x - 2$

working towards a solution ***(A1)***

eg $\cos x = 0$, combining like terms

$x = \frac{\pi}{2}$ ***A1 N0***

Note: Do not award final ***A1*** if additional values are given.

[6 marks]

Examiners report

a. In part (a), most candidates were able to correctly find the derivative of the function.

b. In part (b), many candidates did not understand the significance of the axis of symmetry and the known point (0, 5), and so were unable to find $g(4)$ using symmetry. A few used more complicated manipulations of the function, but many algebraic errors were seen.

c. In part (c), a large number of candidates were able to simply write down the correct value of h, as intended by the command term in this question. A few candidates wrote down the incorrect negative value. Most candidates attempted to substitute the x and y values of the known point correctly into the function, but again many arithmetic and algebraic errors kept them from finding the correct value for a.

d. Part (d) required the candidates to find the derivative of g, and to equate that to their answer from part (a). Although many candidates were able to simplify their equation to $\cos x = 0$, many did not know how to solve for x at this point. Candidates who had made errors in parts (a) and/or (c) were still able to earn follow-through marks in part (d).

Write down the value of

a(i).(i) $\log_3 27$; [1]

a(ii).(ii) $\log_8 \frac{1}{8}$; [1]

a(iii). (iii) $\log_{16}4$. [1]

b. Hence, solve $\log_3 27 + \log_8 \frac{1}{8} - \log_{16} 4 = \log_4 x$. [3]

Markscheme

a(i). (i) $\log_3 27 = 3$ ***A1 N1***

[1 mark]

a(ii). (ii) $\log_8 \frac{1}{8} = -1$ ***A1 N1***

[1 mark]

a(iii). (iii) $\log_{16} 4 = \frac{1}{2}$ ***A1 N1***

[1 mark]

b. correct equation with **their** three values ***(A1)***

eg $\frac{3}{2} = \log_4 x,\ 3 + (-1) - \frac{1}{2} = \log_4 x$

correct working involving powers ***(A1)***

eg $x = 4^{\frac{3}{2}},\ 4^{\frac{3}{2}} = 4^{\log_4 x}$

$x = 8$ ***A1 N2***

[3 marks]

Examiners report

a(i). [N/A]
a(ii). [N/A]
a(iii). [N/A]
b. [N/A]

Let $f(x) = 3\tan^4 x + 2k$ and $g(x) = -\tan^4 x + 8k\tan^2 x + k$, for $0 \leqslant x \leqslant 1$, where $0 < k < 1$. The graphs of f and g intersect at exactly one point. Find the value of k.

Markscheme

discriminant $= 0$ (seen anywhere) ***M1***

valid approach ***(M1)***

eg $f = g,\ 3\tan^4 x + 2k = -\tan^4 x + 8k\tan^2 x + k$

rearranging their equation (to equal zero) ***(M1)***

eg $4\tan^4 x - 8k\tan^2 x + k = 0,\ 4\tan^4 x - 8k\tan^2 x + k$

recognizing LHS is quadratic ***(M1)***

eg $4(\tan^2 x)^2 - 8k\tan^2 x + k = 0,\ 4m^2 - 8km + k$

correct substitution into discriminant ***A1***

eg $(-8k)^2 - 4(4)(k)$

correct working to find discriminant or solve discriminant $= 0$ ***(A1)*** *eg* $64k^2 - 16k,\ \frac{-(-16)\pm\sqrt{16^2}}{2\times 64}$

correct simplification ***(A1)***

*eg*x $16k(4k-1),\ \frac{32}{2\times 64}$

$k = \frac{1}{4}$ ***A1*** ***N2***

[8 marks]

Examiners report

There was a minor issue with the domain of the function, but this did not affect any candidate. The question was amended for publication.

Most candidates recognized the need to set the functions equal to each other and many rearranged the equation to equal zero. Few students then recognized the quadratic form and the need to find the discriminant. Those who did use the discriminant generally completed it correctly.